المملكة العربيـة السعوديـة
وزارة التعليم العالـــــي
جامعــــــــــة أم القرى
كلية العـــــلوم التطبيقية
قسم العـــــلوم الرياضية

مسألة بينارد لطبقة مسامية أفقية يتخللها مائع موصل في وجود كلٍ من المجال المغناطيسي وقوى التدوير

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بحث تكميلي لمتطلبات الحصول على درجة الماجستير في الرياضيات التطبيقية

١٤٢٤هـ/ ٢٠٠٣م

بسم الله الرحمن الرحيم

(ربنا لا تؤاخذنا إن نسينا أو أخطأنا)

صدق الله العظيم

شكر وتقدير

الحمد لله الذي أكمل لنا الدين وأتم علينا النعمة وجعل أمتنا ولله الحمد، خير أمة، وبعث فينا رسولاً منا يتلو علينا آياته ويزكينا ويعلمنا الكتاب والحكمة، أحمده على نعمه الجَمة، وأشهد أن لا إله إلا الله وحده لا شريك له شهادة تكون لمن أعتصم بها خير عصمه، وأشهد أن محمداً عبده ورسوله أرسله للعالمين رحمه، وفرض عليه بيان ما أنزل إلينا فأوضح لنا كل الأمور المهمة، وخصه بجوامع الكلم فربما جمع شتات الحكم والعلوم في كلمه أو شطر كلمه، صلى الله عليه وعلى آله و أصحابه صلاة تكون لنا نوراً من كل ظلمه، وسلم تسليما. أما بعد

فالشكر ترجمان النية ولسان الطوية وشاهد الإخلاص وعنوان الاختصاص وأنا في هذا المقام أتقدم بشكر كأنفاس الأحبار أو أنفاس الرياض غب الأمطار إلى كل من شجعني وآزرني خلال مشواري الدراسي وأخص بالشكر سعادة الأستاذ الدكتور عبد الله أحمد عبد الله والذي كان لجهوده المبذولة بصمه واضحة في إظهار البحث بهذه الصورة فله مني جزيل الشكر وعظيم الامتنان كما أتقدم بالشكر الجزيل إلي جميع أساتذتي الكرام ، كما أنه لا يفوتني أن أقدم شكر خاص إلى مدينة الملك عبد العزيز للعلوم و التقنية لدعمها الجزئي للبحث.

المقام هنا مقام امتنان و عرفان... فباقة شكر أقدمها إلي جميع أفراد عائلتي على كل ما قدموه لي من دعم وجهود لا تنسى وأخص بالشكر الوالدة الغالية التي كانت و ما زالت الشمعة التي تضئ لي حياتي وزوجي الذي لم يدخر جهدا في سبيل نجاحي وتقدمي العلمي.

الإهداء

أهدي قلائد أفكاري فرائدها   در نهضت به من أبحر عمق
يضمها ورق لولا محاسنه   ما لقبوا الفضة البيضاء بالورق

لك أبي ...

يا شمس أضاءت خطا دربي

يا عين سهرت دوماً من اجلي

يا طيف أبداً لم يفارقني

يا روح أحسها دوما معي

لك في السماء يا أبي أهدي

حبي ودعائي وخلاصة جهدي

لك أمي ...

يا درة مكنونـــــــــــــــة
من العيب خلـــــــــت

يا امرأة حنونـــــــــــــة

من الحب تكونـــــــت

يا محيطات الشمعـــــــدان

يا وصلة للجـــــــنان

يا.. يا.. أمي أهديـــــــك

بعض ما خطه البـنان

لك يا زوجي...

ونصفي الثـــــــاني

والذي من الوقت والجهد كم أعطاني

وبا لصبر و العزم شجعني و قواني

حتى تحققت معه كل الأماني

فلك أهدي شهادتي وأحلامي

ملخص الدراسة

يدرس هذا البحث مسألة بينارد لطبقة مسامية أفقية يتخللها مائع لزج وغير قابل للإنضغاط وموصل للحرارة و الكهرباء تحت تأثير مجال مغناطيسي ثابت موازي لمجال الجاذبية الأرضية، وقوى تدوير حول محور موازٍ لنفس الإتجاه ويتم تسخين هذه الطبقة من الأسفل. وقد درست المسألة عندما تكون العلاقة بين شدة المجال المغناطيسي والحث المغناطيسي علاقة خطية و غير خطية.

وتتكون الرسالة من أربعة فصول:

الفصل الأول: مقدمة.

ويشتمل هذا الفصل على مقدمة عامة للموضوع مجال الدراسة وعرض لأهم الدراسات السابقة المتعلقة بموضوع الدراسة.

الفصل الثاني: تقريب شيبيشيف لحل مسائل القيمة الحدية.

يحتوي هذا الفصل على عرض لطريقة شيبيشيف لحل مسائل القيمة الحدية، وهي الطريقة العددية المستخدمة لحل المسألة عددياً، مع ذكر مثال يوضح طريقة تطبيقها.

ويشمل هذا الفصل أيضا شرح مبسط لطريقة التقسيم الذهبي(Golden section search) لإيجاد القيمة الصغرى لدالة ما، حيث تم تطبيقها لإيجاد قيمة عدد ريلي.

الفصل الثالث: مسألة بينارد لطبقة مسامية أفقية يتخللها مائع موصل في وجود كلٍ من المجال المغناطيسي وقوى التدوير.

يشتمل هذا الفصل على دراسة مسألة البحث عندما تكون العلاقة بين شدة المجال المغناطيسي و الحث المغناطيسي خطية وذلك عندما تكون الطبقة مسخنة من الأسفل وتمت مناقشة:

- عدم الاستقرار الثابت (Stationary convection) .

- عدم الاستقرار المفرط(Overstability) .

وقد تم دراسة المسألة بطريقة تحليلية وعددية في حالة كون الحدود العلوية والسفلية من النوع الحر بالنسبة لعدم الاستقرار الثابت وبطريقة عددية فقط في حالة كون الحدود العلوية والسفلية من النوع الصلب. أما بالنسبة لعدم الاستقرار المفرط فقد تم دراسة المسألة عددياً فقط عندما تكون الشروط الحدية من النوع الحر أو الصلب.

أهم النتائج التي تم التوصل إليها:

(١) إن الوسط المسامي يعمل على زيادة ثبات المائع.

(٢) إن الوسط المسامي يؤخر ظهور التعارض بين الحقل المغناطيسي والدوران الذي أشار إليه شاندرازكر في كتابه (١٩٦٠م).

(٣) إن المائع يصبح أكثر ثباتا عندما تقل نفاذية الوسط المسامي.

(٤) إن القيمة الحرجة لرقم شاندرازكر Q تزداد عندما تقل نفاذية الوسط المسامي.

الفصل الرابع: مسألة بينارد لطبقة مسامية أفقية يتخللها مائع مغناطيسي غير خطي في وجود كلٍ من المجال المغناطيسي وقوى التدوير.

يناقش هذا الفصل مسألة البحث عندما تكون العلاقة بين شدة المجال المغناطيسي والحث المغناطيسي علاقة غير خطية وذلك عند تسخين الطبقة من الأسفل، ونوقش كلاً من:

- عدم الاستقرار الثابت.
- عدم الاستقرار المفرط.

وقد تم دراسة المسألة بطريقة تحليلية وأخرى عددية في حال كون الشروط الحدية من النوع الحر، وبطريقة عددية فقط في حال كون الشروط الحدية من النوع الصلب.

<u>أهم النتائج التي تم التوصل إليها:</u>

(١) إن العلاقة غير الخطية بين شدة المجال المغناطيسي والحث المغناطيسي ليس لها تأثير على عدم الاستقرار الثابت، وإنما تؤثر فقط على بداية عدم الاستقرار المفرط.

(٢) إن المائع يصبح أكثر ثباتاً عندما تقل نفاذية الوسط المسامي.

(٣) إن القيمة الحرجة لرقم شاندرازكر Q تزداد عندما تقل نفاذية الوسط المسامي.

(٤) إن العلاقة غير الخطية تعمل على زيادة ثبات المائع.

وزارة التعليم العالي
جامعة أم القرى
كلية العلوم التطبيقية

بسم الله الرحمن الرحيم

محضر مناقشة رسالة (ماجستير)

الحمد لله والصلاة والسلام على رسول الله صلى الله عليه وسلم وعلى آله وصحبه ومن والاه ففي تمام الساعة ٩:٠٠ من يوم السبت الموافق ٧/٤/١٤٢٤ اجتمعت اللجنة المشكلة بقرار مجلس كلية رقم .... وفي جلسته ..... بتاريخ ..../..../........ والمكونة من أصحاب السعادة :

م	الاسم	الجهة التابع لها	وضع المناقش في اللجنة
١	د. خالد بن سعد كحيمر	جامعة الملك عبدالعزيز	مناقش خارجي
٢	أ.د. محمد عبدالإله أحمد عبده	جامعة أم القرى	مناقش داخلي
٣	أ.د. عبدالله أحمد عبدالله	جامعة أم القرى	مشرف ومقرر اللجنة

لمناقشة الرسالة المقدمة لنيل درجة الماجستير في العلوم الرياضية ..
من الطالب / الطالبة فايزة بنت محمد سليم الحجيلي
تخصص رياضيات تطبيقية بكلية العلوم التطبيقية بمكة المكرمة بجامعة أم القرى .
بعنوان [illegible]

وبعد الانتهاء اللجنة من المناقشة في الساعة ١١:٤٥ والتداول فيما بينها .
وبناء على موقف الطالب / الطالبة أثناء المناقشة .
أوصت اللجنة بمنح درجة الماجستير في العلوم الرياضية بتقدير ممتاز بدرجة ٩١,٢٥ المرجحة . والله الموفق .
ما اكتسبه من تقديرات بالسنة المنهجية ٤,٢٠/٥,٠٠
ما اكتسبه من الساعات المعتمدة المطلوبة لهذه الدرجة ٢٤ ساعة

ملاحظات
تقدير تقييم الرسالة ٦٧,٢
المجموع الكلي ٩١,٢٥

م	الاسم	الدرجة	التوقيع
١	د. خالد بن سعد كحيمر	٩٦	
٢	أ.د. محمد عبدالإله أحمد عبده	٩٥	
٣	أ.د. عبدالله أحمد عبدالله	٩٧	
	مجموع الدرجات	٢٨٨	
	المتوسط	٩٦	

رئيس قسم العلوم الرياضية

د. عبدالفتاح بن قاري بخاري

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شرف...

Yamamoto & Iwamura (1976) and Rudraiah, et al. (1980) showed that the Brinkman model is valid up to magnitude of $\kappa_1/d^2 \cong 10^{-4}$ or 10^{-3}, where κ_1 is the permeability of porous medium.

A numerical study of buoyancy-driven two-dimensional convection in a fluid saturated horizontal porous layer confined between two impermeable walls and heated isothermally from below has been studied by Georgiadis & Catton (1986). Kladias & Prasad (1990) studied numerically thermoconductive instabilities in horizontal porous layers heated from below using the Brinkman-Forchheimer-Darcy model. They showed that there are four flow regimes in the case of free convection: conduction, stable convection, periodic oscillatory and randomly oscillatory convection. Jan (2000) used Brinkman model to investigate the convective instability of a horizontal porous layer permeated by a conducting magnetic field. This problem has been extended by Abdullah (2000) for a non-linear magnetic fluid.

The effect of the earth's magnetic field on the stability of a layer of porous medium is of interest in geophysics particularly in the study of the earth's core where the earth's mantle, which consists of conducting fluid, behaves like a porous medium which can become convectively unstable as a result of diffusion. Another application of the results of flow through a porous medium in the presence of a magnetic field lies in the study of the stability of a convective flow in the geothermal region.

This thesis studies convective instability in a horizontal porous layer permeated by an incompressible, thermally and electrically conducting fluid using Brinkman model in the presence of a uniform vertical magnetic field and a uniform vertical rotation for both stationary and overstability cases. The problem is investigated for the cases of linear and non-linear relation between the magnetic field and the magnetic induction in chapter three and chapter four respectively. It is an extension work of Jan (2000) and Abdullah (2000) for the linear and non-linear cases respectively. Analytical solutions were discussed when both boundaries are free and numerical results were obtained for the cases of free and rigid boundaries.

The numerical computations were performed using expansions of Chebyshev polynomials. This method was first introduced by Lanczos (1938) and Clenshaw (1957) and has been developed and extensively applied to ordinary differential equations by Fox (1962), Fox & Parker (1968), Orszag (1971, 1971), Orszag & Kells (1980), Davis, et al (1988, 1988) and others. Nasr, et al (1989) used El-gendi method to obtain numerical solutions for the Falkner-Skan equation. Hassanien, et al. (1996) used Chebyshev polynomials to obtain numerical solutions of the boundary layer flow of a micropolar fluid in the vicinity of an axisymmetric stagnation point on a circular cylinder. Hassanien, et al. (1998,1998) used expansion of Chebyshev polynomials to obtain numerical solutions for some problems of heat transfer. The

Chebyshev method for boundary value problem will be described in detail in chapter two.

Chapter Two

Chebyshev approximation for boundary value problems

Previous works have shown that expansions in Chebyshev polynomials are better suited to the solutions of hydrodynamic stability problems than expansions in other sets of orthogonal functions. Results of high accuracy are obtained when using Chebyshev approximation to compute solutions of boundary value problems.

In this chapter we shall present the simple definition of Chebyshev polynomials and obtain some of their properties.

Definition:

The nth degree Chebyshev polynomial of the first kind is denoted by $T_n(x)$ and is defined in the interval [-1,1] by

$$T_n(\cos\theta) = \cos n\theta. \qquad n \geq 1 \ , \quad 0 \leq \theta \leq \pi \qquad (2.1)$$

From the trigonometric identity

$$\cos(n+1)\theta + \cos(n-1)\theta = 2\cos\theta\cos n\theta$$

we obtain the relation

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x) \qquad n \geq 1 \qquad (2.2)$$

where $x = \cos\theta$. Clearly, from the definition of $T_n(x)$, we have

$$|T_n(x)| \leq 1. \qquad -1 \leq x \leq 1 \qquad (2.3)$$

In terms of the variable x, we may write (2.1) as

$$T_n(x) = \cos(n\cos^{-1}(x)) \tag{2.4}$$

From which we can show that

$$\begin{aligned} T_n'(x) &= \frac{n\sin(n\cos^{-1}(x))}{\sqrt{1-x^2}} \\ &= \frac{n\sin n\theta}{\sin\theta} \end{aligned} \tag{2.5}$$

where $T_n'(x)$ is a polynomial of degree(n-1). By evaluating the expression (2.5) at $x = \pm 1$, it can be shown that

$$T_n'(1) = n^2 \quad , \qquad T_n'(-1) = (-1)^{n-1} n^2 \tag{2.6}$$

and from (2.4)

$$T_n(1) = 1 \quad , \qquad T_n(-1) = (-1)^n . \tag{2.7}$$

The Chebyshev polynomials are orthogonal over $[-1,1]$ with respect to the weight function $(1-x^2)^{-1/2}$. To see that consider the trigonometric integral

$$\int_0^\pi \cos n\theta \cos m\theta \, d\theta = C_n \delta_{mn} . \tag{2.8}$$

where $C_0 = \pi$, $C_n = \pi/2$, $n \neq 0$ and δ_{mn} is the kronecker delta defined by

$$\delta_{mn} = \begin{cases} 1 & \text{if} \quad m = n, \\ 0 & \text{if} \quad m \neq n. \end{cases}$$

With the usual change of variable, $x = \cos\theta$, equation (2.8) becomes

$$\int_{-1}^{1} \frac{T_n(x) T_m(x)}{\sqrt{1-x^2}} dx = C_n \delta_{mn} . \tag{2.9}$$

Let $y(x)$ be an infinite differentiable function, then its Chebyshev expansion in [−1, 1] has the form

$$y(x) = \sum_{n=0}^{\infty} a_{n+1} T_n(x). \qquad (2.10)$$

From the orthogonality property of Chebyshev polynomial and equation (2.10), we obtain

$$\begin{aligned}\int_{-1}^{1} \frac{y(x)T_m(x)}{\sqrt{1-x^2}} dx &= \sum_{n=0}^{\infty} a_{n+1} \int_{-1}^{1} \frac{T_n(x)T_m(x)}{\sqrt{1-x^2}} \\ &= \sum_{n=0}^{\infty} a_{n+1} C_m \delta_{mn} \\ &= \sum_{n=0}^{\infty} a_{m+1} C_m .\end{aligned}$$

Thus

$$a_{m+1} = \frac{1}{C_m} \int_{-1}^{1} y(x) T_m(x) (1-x^2)^{-1/2} dx. \qquad m \geq 0$$

The Chebyshev expansion of the derivative $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} a_{n+1} T_n'(x) \qquad (2.11)$$

where $T_n'(x)$ is a polynomial of degree $(n-1)$. Now we write $T_n'(x)$ as an expansion of Chebyshev polynomials [see Abdullah & Lindsay (1991)]

i.e. $$T_n'(x) = \sum_{s=0}^{\infty} B_{s+1,n+1} T_s(x) \qquad n \geq 0 \qquad (2.12)$$

where $B_{s+1,\,n+1} = 0$ if $s \geq n$ since $T_n'(x)$ is of degree $(n-1)$.

From equation (2.12)

$$\int_{-1}^{1}\frac{T_n'(x)T_j(x)}{\sqrt{1-x^2}}dx=\sum_{s=0}^{\infty}B_{s+1,n+1}\int_{-1}^{1}\frac{T_s(x)T_j(x)}{\sqrt{1-x^2}}dx.$$

Using (2.9) we obtain

$$\int_{-1}^{1}\frac{T_n'(x)T_j(x)}{\sqrt{1-x^2}}dx=\frac{\pi}{2}\sum_{s=0}^{\infty}B_{s+1,\,n+1}\begin{cases}\delta_{sj} & j>0,\\ 2\delta_{sj} & j=0.\end{cases}$$

Hence

$$\begin{aligned}B_{j+1,n+1}&=\frac{2}{\pi}\int_{-1}^{1}\frac{T_n'(x)T_j(x)}{\sqrt{1-x^2}}dx \qquad && j\geq 1,\\ B_{1,\,n+1}&=\frac{1}{\pi}\int_{-1}^{1}\frac{T_n'(x)}{\sqrt{1-x^2}}dx. && j=0.\end{aligned}$$

From (2.5) we obtain

$$\begin{aligned}B_{j+1,n+1}&=\frac{2}{\pi}\int_{0}^{\pi}\frac{n\sin(n\theta)\cos(j\theta)}{\sin\theta}d\theta\\ &=\frac{n}{\pi}\left[\int_{0}^{\pi}\frac{\sin(n+j)\theta}{\sin\theta}d\theta+\int_{0}^{\pi}\frac{\sin(n-j)\theta}{\sin\theta}d\theta\right]\\ &=n\left[I_{n+j}+I_{n-j}\right]\end{aligned}$$

and

$$\begin{aligned}B_{1,n+1}&=\frac{1}{\pi}\int_{0}^{\pi}\frac{n\sin(n\theta)}{\sin\theta}d\theta\\ &=nI_n,\end{aligned}$$

where

$$I_n=\frac{1}{\pi}\int_{0}^{\pi}\frac{\sin n\theta}{\sin\theta}d\theta.$$

Now since

$$I_n = \begin{cases} 0 & \text{if} \quad n \text{ is even,} \\ 1 & \text{if} \quad n \text{ is odd.} \end{cases}$$

then

$$B_{rk} = \begin{cases} 0 & \text{if} \quad r+k \quad \text{is even,} \\ (k-1)(2-\delta_{1r}) & \text{if} \quad r+k \quad \text{is odd.} \end{cases} \tag{2.13}$$

From (2.11) and (2.12) we have

$$\frac{dy}{dx} = \sum_{n=0}^{\infty}\sum_{j=0}^{\infty} B_{j+1,n+1} a_{n+1} T_j(x)$$

and

$$\frac{d^r y}{dx^r} = \sum_{n=0}^{\infty}\sum_{j=0}^{\infty} B^r_{j+1,\,n+1} a_{n+1} T_j(x).$$

The matrix $\underline{B}$ is called the derivative matrix.

Application:

To illustrate the method of solving differential equations using expansions of Chebyshev polynomials, let us consider the problem of determining the eigenvalues σ of the Benard problem under the influence of both magnetic field and rotation. The relative equations of this problem are:

$$\begin{aligned} \sigma\xi &= L\xi + DJ + \sqrt{T}\,Dw, \\ \sigma P_m J &= LJ + QD\xi, \\ \sigma Lw &= L^2 w - a^2\sqrt{R}\,\theta + L(Db) - \sqrt{T}\,D\xi, \\ \sigma P_m b &= Lb + QDw, \\ \sigma P_r \theta &= L\theta + \sqrt{R}\,w. \end{aligned} \tag{2.14}$$

where P_m, P_r are the magnetic and viscous Prandtle numbers, a is the wave number, R is the Rayleigh number, Q is the Chandrasekhar number, T is the Taylor number, $L = D^2 - a^2$, $D = \frac{\partial}{\partial z}$ and w, b, θ, J and ξ are the third components of velocity, magnetic field, temperature, current density and vortisity. To obtain accurate results and exclude any numerical divergence it is convenient to reduce the order of higher order terms, so let us reduce the order of the terms $L^2 w$ and $L(Db)$ in

equation $(2.14)_3$. If we assume that $\phi = Lw$ and use equation $(2.14)_4$ to substitute for $L(Db)$, then equation $(2.14)_3$ becomes

$$\sigma\,(\phi - P_m Db) = L\phi - a^2\sqrt{R}\,\theta - QD^2 w - \sqrt{T}\,D\xi \qquad (2.15)$$

and the relative equations (2.14) become

$$\begin{aligned}
\sigma\xi &= L\xi + DJ + \sqrt{T}\,Dw,\\
\sigma P_m J &= LJ + QD\xi,\\
\sigma\,(\phi - P_m Db) &= L\phi - a^2\sqrt{R}\,\theta - QD^2 w - \sqrt{T}\,D\xi,\\
\sigma\,P_m b &= Lb + QDw,\\
\sigma P_r\theta &= L\theta + \sqrt{R}\,w,\\
\sigma\times 0 &= Lw - \phi.
\end{aligned} \qquad (2.16)$$

The system of equations (2.16) can be solved using free or rigid boundaries. For free boundaries the conditions are

$$w = 0,\ \theta = 0,\ \phi = 0,\ J = 0,\ Db = 0 \text{ and } D\xi = 0, \quad 0 \le z \le 1 \qquad (2.17)$$

and for rigid boundaries the conditions are

$$w = 0,\ \theta = 0,\ Dw = 0,\ J = 0,\ b = 0 \text{ and } \xi = 0. \quad 0 \le z \le 1 \qquad (2.18)$$

At this stage we express all the variables of the problem in terms of Chebyshev polynomials in the following way:

$$(w,\theta,\phi,b,\xi,J)=\sum_{n=0}^{\infty}(a_{n+1},b_{n+1},c_{n+1},d_{n+1},e_{n+1},f_{n+1})T_n(z). \tag{2.19}$$

Since the Chebyshev polynomials are defined in the interval $[-1,1]$ and our problem is defined in the interval $[0,1]$, then we may introduce the variable x such that

$$x=2z-1.$$

Thus

$$\begin{aligned}
D(\ )&=2\frac{d(\ )}{dx}=2B,\\
D^2(\ )&=4\frac{d^2(\ )}{dx^2}=4B^2,\\
L(\ )&=D^2(\ )-a^2(\ )=\frac{4d^2(\ )}{dx^2}-a^2(\ )=4\left[\frac{d^2(\ )}{dx^2}-\left(\frac{a}{2}\right)^2(\ )\right]=4V,
\end{aligned}$$

where $V(\ )=\dfrac{d^2}{dx^2}(\ )-\left(\dfrac{a}{2}\right)^2(\ )$ and $\underline{B}$ is the derivative matrix defined in (2.13). Apply (2.19) into the governing equations (2.16) and the boundary conditions (2.17) and (2.18) then the governing equations become

$$\begin{aligned}
\sigma\, e_{n+1}&=4Ve_{n+1}+2Bf_{n+1}+2B\sqrt{T}\,a_{n+1}\\
\sigma P_m f_{n+1}&=4Vf_{n+1}+2BQe_{n+1}\\
\sigma\,(c_{n+1}-2P_mBd_{n+1})&=4Vc_{n+1}-a^2\sqrt{R}\,b_{n+1}-4QB^2a_{n+1}-2B\sqrt{T}\,e_{n+1}\\
\sigma P_m d_{n+1}&=4Vd_{n+1}+2BQa_{n+1}\\
\sigma P_r b_{n+1}&=4Vb_{n+1}+\sqrt{R}\,a_{n+1}\\
\sigma\times 0&=4Va_{n+1}-c_{n+1},
\end{aligned} \tag{2.20}$$

the free boundary conditions become

$$\sum_{n=0}^{\infty} a_{n+1} T_n(x) = \sum_{n=0}^{\infty} b_{n+1} T_n(x) = \sum_{n=0}^{\infty} c_{n+1} T_n(x) = 0,$$

$$\sum_{n=0}^{\infty} d_{n+1} T_n'(x) = \sum_{n=0}^{\infty} e_{n+1} T_n'(x) = \sum_{n=0}^{\infty} f_{n+1} T_n(x) = 0, \qquad -1 \le x \le 1 \tag{2.21}$$

and the rigid boundary conditions become

$$\sum_{n=0}^{\infty} a_{n+1} T_n(x) = \sum_{n=0}^{\infty} b_{n+1} T_n(x) = \sum_{n=0}^{\infty} a_{n+1} T_n'(x) = 0$$

$$\sum_{n=0}^{\infty} d_{n+1} T_n(x) = \sum_{n=0}^{\infty} e_{n+1} T_n(x) = \sum_{n=0}^{\infty} f_{n+1} T_n'(x) = 0. \qquad -1 \le x \le 1 \tag{2.22}$$

Equations (2.20) can be written in the form

$$\sigma E \underline{X} = A \underline{X} \tag{2.23}$$

where

$$E = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & P_m I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & -2P_m B & 0 & 0 \\ 0 & 0 & 0 & P_m I & 0 & 0 \\ 0 & 0 & 0 & 0 & P_r I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 4V & 2B & 0 & 0 & 0 & 2B\sqrt{T} \\ 2BQ & 4V & 0 & 0 & 0 & 0 \\ -2B\sqrt{T} & 0 & 4V & 0 & -a^2\sqrt{R}\, I & -4QB^2 \\ 0 & 0 & 0 & 4V & 0 & 2BQ \\ 0 & 0 & 0 & 0 & 4V & \sqrt{R}\, I \\ 0 & 0 & -I & 0 & 0 & 4V \end{bmatrix}$$

and $\underline{X} = [e_{n+1} \quad f_{n+1} \quad c_{n+1} \quad d_{n+1} \quad b_{n+1} \quad a_{n+1}]^T$.

Using (2.6) and (2.7), the free boundary conditions (2.21) become

$$\begin{aligned}&\sum_{n=0}^{\infty}(\pm1)^n a_{n+1}=0, \quad \sum_{n=0}^{\infty}(\pm1)^n b_{n+1}=0, \quad \sum_{n=0}^{\infty}(\pm1)^n c_{n+1}=0,\\ &\sum_{n=0}^{\infty}n^2(\pm1)^{n-1} d_{n+1}=0, \quad \sum_{n=0}^{\infty}n^2(\pm1)^{n-1} e_{n+1}=0, \quad \sum_{n=0}^{\infty}(\pm1)^n f_{n+1}=0.\end{aligned} \tag{2.24}$$

and the rigid boundary conditions (2.22) become

$$\begin{aligned}&\sum_{n=0}^{\infty}(\pm1)^n a_{n+1}=0, \quad \sum_{n=0}^{\infty}(\pm1)^n b_{n+1}=0, \quad \sum_{n=0}^{\infty}n^2(\pm1)^{n-1} a_{n+1}=0,\\ &\sum_{n=0}^{\infty}(\pm1)^n d_{n+1}=0, \quad \sum_{n=0}^{\infty}(\pm1)^n e_{n+1}=0, \quad \sum_{n=0}^{\infty}n^2(\pm1)^{n-1} f_{n+1}=0.\end{aligned} \tag{2.25}$$

Notice that each element of the matrix A and E represents a square matrix. The order of this matrix depends on the number of Chebyshev polynomials used for the required accuracy. In general the boundary conditions (2.24) or (2.25) should be inserted in the first and second rows of each diagonal element of the matrix A. The corresponding rows in the other elements of A and E must be set to zero.

Each diagonal element of A and E corresponds to a variable element in $\underline{X}$, so in this problem we can see from equations (2.20) that the first diagonal element corresponds to ξ, the second corresponds to J, the third corresponds to ϕ, the fourth corresponds to b, the fifth corresponds to θ and the sixth corresponds to w. Thus the first row of each diagonal element of A contains the condition of the corresponding variable at $x=1$ whereas the second row contains the condition of that variable at $x=-1$.

If the boundary conditions in (2.24) or (2.25) are of the form $\sum_{n=0}^{\infty}(\pm 1)^n \alpha_{n+1} = 0$, then the first row of the corresponding matrix is of the form $(1,1,1,........)$ and the second row is of the form $(1,-1,1,-1,.....)$. However if the boundary conditions in (2.24) or (2.25) are of the form $\sum_{n=0}^{\infty} n^2(\pm 1)^n \alpha_{n+1} = 0$, then the first row is of the form (0, 1, 4, 9, 16, ...) and the second row is of the form $(0,-1,4,-9,16,...)$

In the case of rigid boundaries, the condition $\phi = 0$ is replaced by $Dw = 0$, so we remove the condition $\phi = 0$ which is written in the diagonal element of the third row of the matrix A and insert the condition $Dw = 0$ in the same row but in the column corresponds to the variable w, i.e. the sixth column in this problem.

In practice we cannot use infinite series of Chebyshev polynomials and we have to rely on a finite approximation of suitable accuracy, so we look for an approximation of the form

$$Y(x) = \sum_{n=0}^{N} a_{n+1} T_n(x)$$

where N is the number of Chebyshev polynomials required.

In the theory of convection, it is customary to introduce time dependence through $\exp(\sigma t)$ and instability occurs if any eigenvalue σ has a positive real part. The most competitive or " least stable " eigenvalue has the algebraically largest real part. At this stage the eigenvalue problem (2.23) can be solved using a numerical routine F02BJF which is part of a mathematical library called numerical algorithm group (NAG). The Fortran code for this example is listed in Appendix (I).

The golden section search:

To obtain the critical Rayleigh number, R, in equations (2.14) we need to minimize R over the wave number, a, using a minimization method. The method we shall use is called the golden section search. It is an iterative method and it deals with a unimodel function, i.e. a function which has only a single local minimum.

The method depends on a ratio r known to the early Greeks as the golden section ratio and is given by

$$r = \frac{1}{2}\left(\sqrt{5} - 1\right),$$

where $r^2 = 1 - r$. Suppose that $F(x)$ has a single minimum in the interval $[a,b]$. In each step we need to replace this interval by a smaller one that is also contains the minimum point. In fact in each step we need to specify two values of F at two particular points in $[a,b]$. i.e.

$$x = a + r^2(b - a),$$
$$y = a + r(b - a).$$

There are two cases to consider:

Case 1

$$F(x) \geq F(y).$$

Here the minimum of F must be in the interval $[x, b]$ and so in the next step we need to evaluate F at x^*, y^* where

$$\begin{aligned} x^* &= x + r^2(b - x), \\ y^* &= x + r(b - x). \end{aligned}$$

Now

$$\begin{aligned} x^* &= a + r^2(b-a) + r^2\left[b - a - r^2(b-a)\right] \\ &= a + (b-a)\left[2r^2 - r^4\right] \end{aligned}$$

but

$$\begin{aligned} 2r^2 - r^4 &= 1 - (1 - r^2)^2 \\ &= 1 - r^2 \\ &= r, \end{aligned}$$

and so

$$\begin{aligned} x^* &= a + r(b-a) \\ &= y. \end{aligned}$$

Thus $$F(x^*) = F(y).$$

Case 2

Here the minimum of F is in the interval $[a, y]$ and so in the next step we need to evaluate F at x^*, y^* where

$$\begin{aligned} x^* &= a + r^2(y - a), \\ y^* &= a + r(y - a). \end{aligned}$$

Now

$$\begin{aligned} y^* &= a + r[a + r(b-a) - a] \\ &= a + r^2(b-a) \\ &= x. \end{aligned}$$

Thus $$F(y^*) = F(x).$$

In both cases the values of F at one of the new points is already known, In particular, we may verify that

$$y - a = b - x = r(b-a),$$

so that after N iterations of the method, the interval $[a,b]$ is reduced to one of length $r^N(b-a)$ and so if ε is the required accuracy then we need to choose N such that

$$r^N(b-a) < \varepsilon$$

i.e. $$r^N < \frac{\varepsilon}{b-a} \quad \Rightarrow \quad N < \log\left(\frac{\varepsilon}{b-a}\right) \Big/ \log r.$$

Chapter Three

Benard convection in a horizontal porous layer permeated by a conducting fluid in the presence of both magnetic field and Coriolis forces (linear magnetic fluid)

3.1 Mathematical formulation

Consider an infinite horizontal layer occupied by a porous medium permeated by an incompressible, thermally and electrically conducting viscous fluid of density ρ. The fluid is subject to constant gravitational acceleration in the negative x_3 direction. A uniform constant magnetic field is imposed across the layer in the positive x_3 direction and x_1 and x_2 axes are selected from a right-handed system of Cartesian coordinates. The fluid is in rotation about the x_3 axes with a constant angular velocity $\underline{\Omega}$ (see figure 1).

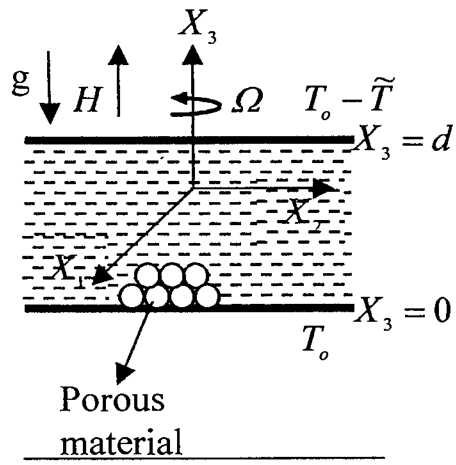

Figure1: Thermal convection of an infinite horizontal layer heated uniformly from below.

In order to fully describe the nature of this model we need to discuss the interaction between electromagnetic and mechanical effects and so we define $\underline{B}, \underline{H}, \underline{E}$ and $\underline{J}$ to be respectively the magnetic induction, the magnetic field, the electric field and the current density. The magnetic variables are required to satisfy the Maxwell equations

$$div\,\underline{B} = B_{i,i}=0,$$
$$curl\,\underline{H}= e_{ijk}H_{k,j}=J_i, \tag{3.1.1}$$
$$curl\,\underline{E} =e_{ijk}E_{k,j}=-\frac{\partial B_i}{\partial t},$$

where the displacement current has been neglected as is customary in this type of problems and where the current density $\underline{J}$ is given by

$$\eta J_i=E_i + e_{ijk}\upsilon_j B_k, \tag{3.1.2}$$

where η is the resistivity (assumed constant) and $\underline{\upsilon}$ is the fluid velocity. The relation between $\underline{B}$ and $\underline{H}$ has form

$$B_i=\mu H_i.$$

On taking the curl of the constitutive law (3.1.2) and replacing the electric field by the Maxwell relation $(3.1.1)_3$, the magnetic induction is now readily seen to satisfy the partial differential equation

$$\frac{\partial \underline{B}}{\partial t}= curl(\underline{\upsilon}\times\underline{B})-\eta\, curl\,\underline{J} \tag{3.1.3}$$

Equation (3.1.3) is now reworked using standard vector identities to yield in sequence

$$\frac{\partial B_i}{\partial t}= e_{ijk}e_{krs}(\upsilon_r B_s)_{,j} -\eta\, e_{ijk}J_{k,j}$$

$$=(\delta_{ir}\delta_{js}-\delta_{is}\delta_{jr})(\upsilon_r B_s)_{,j}-\eta\, e_{ijk}J_{k,j}$$

$$= (\upsilon_i B_j)_{,j} - (\upsilon_j B_i)_{,j} - \eta\, e_{ijk} J_{k,j}$$

$$= B_j \upsilon_{i,j} - \upsilon_j B_{i,j} - \eta\, e_{ijk} J_{k,j}.$$

If we use the conventional notations, namely, θ is absolute temperature and p is hydrostatic pressure then the relevant equation of motion has form

$$\rho \dot{\upsilon}_i = - p_{,i} + \rho\, \nu \nabla^2 \upsilon_i + \rho g_i + (\underline{J} \times \underline{B})_i - \frac{\mu'}{k_1} \upsilon_i + \frac{1}{2} \rho \left| \underline{\Omega} \times \underline{r} \right|^2_{,i} + 2\rho (\underline{\upsilon} \times \underline{\Omega})_i, \tag{3.1.4}$$

where μ' is the dynamic viscosity and k_1 is the permeability of porous medium. The terms $\frac{1}{2} \left| \underline{\Omega} \times \underline{r} \right|^2_{,i}$ and $2(\underline{\upsilon} \times \underline{\Omega})_i$ represent respectively the centrifugal force and the Coriolis acceleration. The term $(\underline{J} \times \underline{B})_i$ in (3.1.4) can be written as

$$(\underline{J} \times \underline{B})_i = (curl\, \underline{H} \times \underline{B})_i$$

$$= e_{ijk} (e_{jrs} H_{s,r}) B_k$$

$$= (\delta_{kr} \delta_{is} - \delta_{ks} \delta_{ir}) H_{s,r} B_k$$

$$= H_{i,k} B_k - H_{k,i} B_k$$

$$= \frac{1}{\mu} (B_{i,k} B_k - B_{k,i} B_k)$$

where $B^2 = B_k B_k$ and since $B^2_{,i} = 2 B_k B_{k,i}$ then

$$(\underline{J} \times \underline{B})_i = \frac{1}{\mu} B_{i,k} B_k - \frac{1}{2\mu} B^2_{,i}$$

If we now make the Boussinesq approximation which assumes that density is constant everywhere in the equation of motion except in its association with the external body force where the density is linearly proportional to temperature,

i.e. $$\rho = \rho_o (1 - \alpha\theta) ,$$

where ρ_o is the density of fluid at temperature T_o, then the governing field equations become,

$$\begin{aligned} \upsilon_{i,i} &= 0, \\ \frac{D\upsilon_i}{Dt} &= -(\frac{P}{\rho_o})_{,i} + \nu\nabla^2 \upsilon_i - g(1-\alpha\theta)\delta_{i3} + \frac{1}{\rho_o \mu} B_{i,k} B_k - \frac{\nu}{k_1}\upsilon_i + 2e_{ijk}\upsilon_j \Omega_k, \\ \frac{D\theta}{Dt} &= k\nabla^2 \theta, \\ \frac{\partial B_i}{\partial t} &= B_j \upsilon_{i,j} - \upsilon_j B_{i,j} - \eta e_{ijk} J_{k,j}, \end{aligned} \tag{3.1.5}$$

together with the Maxwell relations (3.1.1) where $\frac{D(\ )}{Dt}\left(= \frac{\partial(\ )}{\partial t} + \frac{\partial(\ )}{\partial x_i}\frac{\partial x_i}{\partial t} \right)$ is the convected derivative, k is the coefficient of thermal diffusivity, $\nu = \frac{\mu}{\rho_o}$ is the kinematic viscosity, ∇^2 the three dimensional Laplacian operator and

$$P = p + \frac{1}{2\mu} B^2 - \frac{1}{2}\rho_o |\underline{\Omega} \times \underline{r}|^2$$

is the modified pressure.

We may observe that equations (3.1.5) have a steady state solution in which

$$\underline{\upsilon} = 0,$$

$$\theta = \theta(x_3) = T_o - \beta x_3,$$

$$P = P(x_3), \qquad (3.1.6)$$

$$\underline{B} = (0,0,B), \qquad B = \text{constant},$$

$$\underline{J} = 0,$$

where temperatures on the planes $x_3 = 0$ and $x_3 = d$ are T_o and $T_o - \widetilde{T}$ respectively so that $\beta = \dfrac{\widetilde{T}}{d}$, and where the external magnetic field is considered to be normal to the layer of fluid.

Suppose that the initial state described by equations (3.1.6) is slightly perturbed so that

$$\begin{aligned} &\underline{\upsilon} = \underline{0} + \varepsilon^* \underline{\upsilon}^*, \qquad \theta = T_o - \beta x_3 + \varepsilon^* \theta^* \qquad P = P + \varepsilon^* P^*, \\ &\underline{B} = (0,0,B) + \varepsilon^* \underline{b}^*, \qquad \underline{J} = \underline{0} + \varepsilon^* \underline{J}^*, \end{aligned} \qquad (3.1.7)$$

where ε^* is the perturbation parameter and $\underline{\upsilon}^*, \theta^*, P^*, \underline{b}^*$ and $\underline{J}^*$ are respectively the linear perturbation of velocity, temperature, pressure, magnetic induction and current density about their values described in (3.1.6).

We may verify that the linearized versions of equations (3.1.5) are

$$\frac{\partial \upsilon_i^*}{\partial t} = -\left(\frac{P^*}{\rho_o}\right)_{,i} + \nu\nabla^2\upsilon_i^* + g\alpha\theta^*\delta_{i3} + \frac{1}{\rho_o\mu}Bb_{i,3}^* - \frac{\nu}{k_1}\upsilon_i^* + 2e_{ijk}\upsilon_j^*\Omega_k,$$
$$\upsilon_{i,i}^* = 0,$$
$$\frac{\partial \theta^*}{\partial t} - \beta\upsilon_3^* = k\nabla^2\theta^*,$$
$$b_{i,i}^* = 0, \qquad (3.1.8)$$
$$\frac{\partial b_i^*}{\partial t} = B\upsilon_{i,3}^* - \eta e_{ijk}J_{k,j}^*,$$
$$J_i^* = \frac{1}{\mu}e_{ijk}b_{k,j}^*.$$

At this stage we introduce dimensionless variables $\hat{x}_i, \hat{\upsilon}_i, \hat{t}, \hat{J}_i, \hat{\theta}, \hat{P}$ and $\hat{b}_i$ such that

$$x_i^* = d\hat{x}_i, \quad \upsilon_i^* = \frac{k}{d}\hat{\upsilon}_i, \quad t = \frac{d^2}{\nu}\hat{t}, \quad J_i^* = \frac{k\nu\rho_o}{Bd^3}\hat{J}_i,$$
$$\theta^* = \frac{k}{d}\sqrt{\frac{\nu|\beta|}{k\alpha g}}\hat{\theta}, \quad P^* = \frac{k\nu\rho_o}{d^2}\hat{P} \quad \text{and} \quad b_i^* = \frac{k\mu\nu\rho_o}{Bd^2}\hat{b}_i.$$

After this non-dimensionalization, equations (3.1.8) simplify to

$$\upsilon_{i,i} = 0,$$
$$\frac{\partial \upsilon_i}{\partial t} = -P_{,i} + \nabla^2\upsilon_i + \sqrt{R}\theta\delta_{i3} + b_{i,3} - \frac{1}{N}\upsilon_i + \sqrt{T}e_{ijk}\upsilon_j\delta_{k3},$$
$$P_r\frac{\partial \theta}{\partial t} + H\sqrt{R}\upsilon_3 = \nabla^2\theta, \qquad (3.1.9)$$
$$b_{i,i} = 0,$$
$$P_m\frac{\partial b_i}{\partial t} = Q\upsilon_{i,3} - e_{ijk}J_{k,j},$$
$$J_i = e_{ijk}b_{k,j}.$$

where the (^) superscript has been dropped but all the variables are now non-dimensional where the non-dimensional numbers R, N, P_r, P_m, Q and T are given by

$$R = \frac{\alpha g|\beta|d^4}{\nu k}, \qquad N = \frac{k_1}{d^2},$$
$$P_r = \frac{\nu}{k}, \qquad P_m = \frac{\nu\mu}{\eta},$$
$$Q = \frac{B^2 d^2}{\rho_0 \nu \eta}, \qquad T = \frac{4\Omega^2 d^4}{\nu^2}.$$

and

$$H = -\frac{\beta}{|\beta|} = \begin{cases} +1 & \text{when heating from above,} \\ -1 & \text{when heating from below.} \end{cases}$$

Equations $(3.1.9)_5$ and $(3.1.9)_6$ can be combined to give

$$\begin{aligned} P_m \frac{\partial b_i}{\partial t} &= Q\upsilon_{i,3} - e_{ijk}(e_{krs} b_{s,rj}) \\ &= Q\upsilon_{i,3} - (\delta_{ir}\delta_{js} - \delta_{is}\delta_{jr}) b_{s,rj} \\ &= Q\upsilon_{i,3} + b_{i,jj}. \end{aligned} \tag{3.1.10}$$

3.2 The boundary conditions

The fluid is confined between the planes $x_3 = 0$ and $x_3 = 1$ and on these planes we need to specify mechanical, thermal and electromagnetic conditions. Suitable mechanical conditions assume either a rigid or free boundary, suitable thermal conditions assume either a perfectly conducting or an insulating boundary and suitable electromagnetic boundary conditions assume either an electrically insulating or a perfectly conducting boundary.

Mechanical conditions

On a stationary rigid boundary,

$$\upsilon_3 = 0 \qquad \text{on} \qquad x_3 = 0, 1 \ ,$$

i.e. no slip occurs and the normal component of velocity must vanish on these surfaces. This implies that each component of the fluid velocity and all of the x_1, x_2 partial derivatives of each component of the fluid velocity are zero.

On a stationary free boundary $\frac{\partial \upsilon_3}{\partial n} = 0$, where n is the normal to the cylindrical boundary drawn outwards and since

$$\frac{\partial \theta}{\partial n} = \frac{\partial \upsilon_3}{\partial n} \qquad \text{and} \qquad \theta = 0$$

then $\qquad \upsilon_3 = 0.$

Also the x_3 component of velocity is zero and the absence of shear stress requires all partial x_3 derivatives of the tangential velocity components to be zero.

If we define ξ to be the fluid vorticity then $\xi = \text{curl}\ \underline{\upsilon}$ and the mechanical boundary conditions can be expressed by the equations,

$$\xi_3 = 0 \qquad \text{on a rigid boundary,}$$
$$\frac{\partial \xi_3}{\partial x_3} = 0 \qquad \text{on a free boundary}.$$

If the fluid is also incompressible, then υ_3 satisfies the additional property

$$\frac{\partial \upsilon_3}{\partial x_3} = 0 \qquad \text{on a rigid boundary,}$$

$$\frac{\partial^2 \upsilon_3}{\partial x_3^2} = 0 \qquad \text{on a free boundary .}$$

Thermal conditions

At a perfectly conducting boundary, the temperatures of the boundary and impinging fluid match whereas on a perfectly insulating boundary, no heat transfer can take place between the fluid and the surroundings and thus the normal derivative of temperature is zero. In mathematical terms, the possible thermal conditions are

$$\theta = \theta_{ext} \qquad \text{on a conducting boundary,}$$

$$\frac{\partial \theta}{\partial x_3} = 0 \qquad \text{on an insulating boundary,}$$

where θ_{ext} is the temperature of the region exterior to the fluid boundary.

Electromagnetic conditions

On a perfectly insulating electromagnetic boundary, no current can flow to the exterior region and so $J_3 = 0$ and the magnetic field is continous across the boundary with the external magnetic field being derived from a scalar potential since curl $\underline{H} = 0$ in the exterior region. On a stationary perfectly conducting boundary $b_3 = 0$ and there can be no surface components of electric field. Also the surface components of the current density are zero and since $\operatorname{div} \underline{J} = 0$ then $\dfrac{\partial J_3}{\partial x_3} = 0$.

3.3 The eigenvalue problem

We aim to investigate the linear stability of the conduction solution (3.1.6) and with this aim in mind we construct the related eigenvalue problem from equations (3.1.9) and the boundary conditions. When we take the curl of equations $(3.1.9)_2$ and (3.1.10) we obtain

$$\frac{\partial \xi_i}{\partial t} = \nabla^2 \xi_i + \sqrt{R} e_{ijk} \theta_{,j} \delta_{k3} + J_{i,3} - \frac{1}{N} \xi_i + \sqrt{T} \upsilon_{i,3},$$
$$P_m \frac{\partial J_i}{\partial t} = Q \xi_{i,3} + J_{i,jj}. \tag{3.3.1}$$

Taking the curl of equation $(3.3.1)_1$ once again, we obtain

$$\frac{\partial \nabla^2 \upsilon_i}{\partial t} = \nabla^4 \upsilon_i - \sqrt{R}\left(\frac{\partial^2 \theta}{\partial x_3 \partial x_i} - \frac{\partial^2 \theta}{\partial x_j^2} \delta_{i3}\right) + \nabla^2 b_{i,3} - \frac{1}{N} \nabla^2 \upsilon_i - \sqrt{T} \xi_{i,3} \tag{3.3.2}$$

As is the case in many convection problems, vector components parallel to the direction of gravity (i.e. the x_3 direction) play a central role and so it is convenient to introduce the variables w, b, J, ξ and z by the definitions

$$w = \upsilon_3, \quad b = b_3, \quad J = J_3, \quad \xi = \xi_3, \quad z = x_3.$$

Thus the third components of equations (3.3.1), (3.3.2) and (3.1.10) yield

$$\begin{aligned}
&\frac{\partial \xi}{\partial t} = \nabla^2 \xi + \frac{\partial J}{\partial z} - \frac{1}{N}\xi + \sqrt{T}\frac{\partial w}{\partial z}, \\
&P_m \frac{\partial J}{\partial t} = Q\frac{\partial \xi}{\partial z} + \nabla^2 J, \\
&\frac{\partial \nabla^2 w}{\partial t} = \nabla^4 w + \sqrt{R}\left(\frac{\partial^2 \theta}{\partial x_1^2} + \frac{\partial^2 \theta}{\partial x_2^2}\right) + \nabla^2\left(\frac{\partial b}{\partial z}\right) - \frac{1}{N}\nabla^2 w - \sqrt{T}\frac{\partial \xi}{\partial z}, \\
&P_m \frac{\partial b}{\partial t} = Q\frac{\partial w}{\partial z} + \nabla^2 b.
\end{aligned} \tag{3.3.3}$$

Now we look for a solution of the form

$$\Phi = \Phi(z)\ \exp[i(nx + my) + \sigma t].$$

where n, m are the wave numbers of the harmonic disturbance and σ is the growth rate. Thus equations (3.3.3) become

$$\begin{aligned}
&\sigma \xi = L\xi + DJ - \frac{1}{N}\xi + \sqrt{T}\ Dw \\
&\sigma P_m J = QD\xi + LJ \\
&\sigma L w = L^2 w - a^2\sqrt{R}\ \theta + L(Db) - \frac{1}{N}Lw - \sqrt{T}\ D\xi \\
&\sigma P_m b = QDw + Lb \\
&\sigma P_r \theta = L\theta - H\sqrt{R}\ \ w
\end{aligned} \tag{3.3.4}$$

where D is the operator $\frac{\partial}{\partial z}$, $a(=\sqrt{n^2+m^2})$ is the wave number and L is the operator $(D^2 - a^2)$. Eliminating J from equation $(3.3.4)_1$ using equation $(3.3.4)_2$, thus

$$[(L - \sigma P_m)(L - \sigma - \frac{1}{N}) - QD^2]\xi = -\sqrt{T}(L - P_m \sigma)Dw. \tag{3.3.5}$$

We may eliminate b, θ and ξ from equation $(3.3.4)_3$ by applying the operator

$(L-\sigma P_m)(L-\sigma P_r)[(L-\sigma P_m)(L-\sigma-\frac{1}{N})-QD^2]$ to equation $(3.3.4)_3$ and

using equation (3.3.5) to obtain a twelfth order ordinary differential equation to

be satisfied by w.

$$
\begin{aligned}
&\sigma^5 P_m P_r Lw-\sigma^4\left\{ P_m[P_m+2P_r(1+P_m)]L^2w-\frac{2P_rP_m^2}{N}Lw \right\}+\sigma^3\left\{ [P_r(P_m+1)^2\right.\\
&+2P_m(P_m+P_r+1)]L^3w-\frac{2P_m}{N}[P_m+P_r(2+P_m)]L^2w-P_mP_r(2QD^2-\frac{P_m}{N^2})Lw\\
&\left.-a^2RHP_m^2w+TP_rP_m^2D^2w \right\}-\sigma^2\left\{ [(P_m+1)^2+2(P_m+P_r+P_mP_r)]L^4w\right.\\
&-\frac{1}{N}[4P_m(1+P_r)+2(P_r+P_m^2)]L^3w+[\frac{P_m^2}{N^2}(1+\frac{2P_r}{P_m})-2QD^2(P_m+P_r+P_mP_r)]L^2w\\
&\left.+P_mP_r[(\frac{2Q}{N}+T(2+\frac{P_m}{P_r}))D^2-a^2R\frac{H}{P_r}(2+P_m)]Lw+a^2\frac{R}{N}HP_m^2w \right\}\\
&+\sigma\left\{ (2P_m+P_r+2)L^5w-\frac{2}{N}(P_r+2P_m+1)L^4w-[2QD^2(1+P_m+P_r)\right.\\
&-\frac{1}{N^2}(2P_m+P_r)]L^3w+[-a^2RH(1+2P_m)+(\frac{2Q}{N}(P_m+P_r)+T(P_r+2P_m))D^2]L^2w\\
&\left.+(2a^2\frac{R}{N}P_mH+Q^2D^4P_r)Lw+a^2RQHP_mD^2w \right\}-L^6w+\frac{2}{N}L^5w+(2QD^2-\frac{1}{N^2})L^4w\\
&+(a^2RH-2\frac{Q}{N}D^2-TD^2)L^3w-(a^2\frac{R}{N}H+Q^2D^4)L^2w-a^2RQHD^2Lw=0.
\end{aligned}
\tag{3.3.6}
$$

3.4 Stability analysis

Here we shall consider the variation of the Rayleigh number as a function of the magnetic parameter Q, the Taylor number T, the porous medium permeability N and the wave number a for the stationary convection case (i.e. when $\sigma = 0$) using the boundary conditions discussed in section {3.2} when the layer is heated from below (i.e. $H = -1$). If $\sigma = 0$ then we deduce from equations (3.3.4) that

$$\begin{aligned}
&L\xi + DJ - \frac{1}{N}\xi + \sqrt{T}\,Dw = 0, \\
&LJ + QD\xi = 0, \\
&L^2 w - a^2\sqrt{R}\,\theta + L(Db) - \frac{1}{N}Lw - \sqrt{T}\,D\xi = 0, \\
&Lb + QDw = 0, \\
&L\theta + \sqrt{R}\,w = 0.
\end{aligned} \tag{3.4.1}$$

Eliminate b from $(3.4.1)_{3,4}$. Thus

$$L^2 w - a^2\sqrt{R}\,\theta - QD^2 w - \frac{1}{N}Lw - \sqrt{T}\,D\xi = 0. \tag{3.4.2}$$

Equation (3.4.2) can be written in the form

$$D^4 w_1 - (2a_1^2 + Q_1 + \frac{1}{N_1})D^2 w_1 + a_1^4 w_1 - a_1^2\sqrt{R_1}\,\theta_1 + \frac{a_1^2}{N_1}w_1 - \sqrt{T_1}\,D\xi_1 = 0.$$

Multiplying by w_2 and integrate to obtain

$$\begin{aligned}
&\int D^2 w_2 D^2 w_1 + (2a_1^2 + Q_1 + \frac{1}{N_1})\int Dw_2 Dw_1 + a_1^4\int w_2 w_1 - a_1^2\sqrt{R_1}\int w_2\theta_1 \\
&+ \frac{a_1^2}{N_1}\int w_2 w_1 - \sqrt{T_1}\int w_2 D\xi_1 = 0.
\end{aligned} \tag{3.4.3}$$

Similarly

$$\begin{aligned}
&\int D^2 w_1 D^2 w_2 + (2a_2^2 + Q_2 + \frac{1}{N_2})\int Dw_1 Dw_2 + a_2^4\int w_1 w_2 - a_2^2\sqrt{R_2}\int w_1\theta_2 \\
&+ \frac{a_2^2}{N_2}\int w_1 w_2 - \sqrt{T_2}\int w_1 D\xi_2 = 0.
\end{aligned} \tag{3.4.4}$$

Also equation $(3.4.1)_1$ can be written in the form

$$D^2\xi_1 - a_1^2\xi_1 + DJ_1 - \frac{1}{N_1}\xi_1 + \sqrt{T_1}Dw_1 = 0.$$

Multiplying by ξ_2 and integrate to obtain

$$\int D\xi_2 D\xi_1 + a_1^2\int \xi_2\xi_1 + \int J_1 D\xi_2 + \frac{1}{N_1}\int \xi_2\xi_1 = \sqrt{T_1}\int \xi_2 Dw_1. \tag{3.4.5}$$

Similarly

$$\int D\xi_1 D\xi_2 + a_2^2\int \xi_1\xi_2 + \int J_2 D\xi_1 + \frac{1}{N_2}\int \xi_1\xi_2 = \sqrt{T_2}\int \xi_1 Dw_2. \tag{3.4.6}$$

From equation $(3.4.1)_2$

$$D^2 J_1 - a_1^2 J_1 + Q_1 D\xi_1 = 0.$$

Multiplying by J_2 and integrate to obtain

$$\int DJ_2 DJ_1 + a_1^2\int J_2 J_1 = Q_1\int J_2 D\xi_1. \tag{3.4.7}$$

Similarly

$$\int DJ_1 DJ_2 + a_2^2\int J_1 J_2 = Q_2\int J_1 D\xi_2. \tag{3.4.8}$$

By using equation (3.4.8) we can eliminate $\int J_1 D\xi_2$ from (3.4.5). Thus

$$\int D\xi_2 D\xi_1 + a_1^2\int \xi_1\xi_2 + \frac{1}{Q_2}\left[\int DJ_1 DJ_2 + a_2^2\int J_1 J_2\right] + \frac{1}{N_1}\int \xi_1\xi_2 = \sqrt{T_1}\int \xi_2 Dw_1. \tag{3.4.9}$$

Similarly, from equations (3.4.6) and (3.4.7) we can eliminate $\int J_2 D\xi_1$. Thus

$$\int D\xi_1 D\xi_2 + a_2^2 \int \xi_1\xi_2 + \frac{1}{Q_1}\left[\int DJ_2 DJ_1 + a_1^2 \int J_2 J_1\right]$$

$$+\frac{1}{N_2}\int \xi_2\xi_1 = \sqrt{T_2}\int \xi_1 Dw_2. \tag{3.4.10}$$

Since

$$\int w_2 D\xi_1 = \int D(w_2\xi_1) - \int \xi_1 Dw_2 = -\int \xi_1 Dw_2,$$

then using equation (3.4.10) we can eliminate $\int w_2 D\xi_1$ from equation (3.4.3).

Thus

$$\int D^2 w_2 D^2 w_1 + (2a_1^2 + Q_1 + \frac{1}{N_1})\int Dw_2 Dw_1 + a_1^4 \int w_2 w_1 - a_1^2\sqrt{R_1}\int w_2\theta_1 + \frac{a_1^2}{N_1}\int w_2 w_1$$
$$+\frac{\sqrt{T_1}}{\sqrt{T_2}}\left\{ \int D\xi_1 D\xi_2 + a_2^2 \int \xi_2\xi_1 + \frac{1}{Q_1}\left[\int DJ_2 DJ_1 + a_1^2 \int J_2 J_1\right] + \frac{1}{N_2}\int \xi_2\xi_1 \right\} = 0.$$

(3.4.11)

Similarly from equations (3.4.4) and (3.4.9) we can eliminate $\int w_1 D\xi_2$. Thus

$$\int D^2 w_1 D^2 w_2 + (2a_2^2 + Q_2 + \frac{1}{N_2})\int Dw_1 Dw_2 + a_2^4 \int w_1 w_2 - a_2^2\sqrt{R_2}\int w_1\theta_2 + \frac{a_2^2}{N_2}\int w_1 w_2$$
$$+\frac{\sqrt{T_2}}{\sqrt{T_1}}\left\{ \int D\xi_2 D\xi_1 + a_1^2 \int \xi_1\xi_2 + \frac{1}{Q_2}\left[\int DJ_1 DJ_2 + a_2^2 \int J_1 J_2\right] + \frac{1}{N_1}\int \xi_1\xi_2 \right\} = 0.$$

(3.4.12)

From equation $(3.4.1)_5$

$$D^2\theta_1 - a_1^2\theta_1 + \sqrt{R_1}\, w_1 = 0.$$

Multiply by θ_2 and integrate to obtain

$$\int D\theta_2 D\theta_1 + a_1^2 \int \theta_2\theta_1 = \sqrt{R_1} \int \theta_2 w_1. \tag{3.4.13}$$

Similarly

$$\int D\theta_1 D\theta_2 + a_2^2 \int \theta_1\theta_2 = \sqrt{R_2} \int \theta_1 w_2. \tag{3.4.14}$$

By using equation (3.4.14) we can eliminate $\int w_2\theta_1$ from equation (3.4.11) to obtain

$$\int D^2 w_2 D^2 w_1 + (2a_1^2 + Q_1 + \frac{1}{N_1}) \int Dw_2 Dw_1 + a_1^4 \int w_1 w_2 - a_1^2 \frac{\sqrt{R_1}}{\sqrt{R_2}} \left[\int D\theta_1 D\theta_2 + a_2^2 \int \theta_1\theta_2\right]$$
$$+ \frac{a_1^2}{N_1} \int w_1 w_2 + \frac{\sqrt{T_1}}{\sqrt{T_2}} \left\{ \int D\xi_1 D\xi_2 + a_2^2 \int \xi_2\xi_1 + \frac{1}{Q_1}\left[\int DJ_2 DJ_1 + a_1^2 \int J_2 J_1\right] + \frac{1}{N_2} \int \xi_2\xi_1 \right\} = 0. \tag{3.4.15}$$

Similarly from equations (3.4.12) and (3.4.13) we can eliminate $\int \theta_2 w_1$. Thus

$$\int D^2 w_1 D^2 w_2 + (2a_2^2 + Q_2 + \frac{1}{N_2}) \int Dw_1 Dw_2 + a_2^4 \int w_1 w_2 - a_2^2 \frac{\sqrt{R_2}}{\sqrt{R_1}} \left[\int D\theta_2 D\theta_1 + a_1^2 \int \theta_2\theta_1\right]$$
$$+ \frac{a_2^2}{N_2} \int w_1 w_2 + \frac{\sqrt{T_2}}{\sqrt{T_1}} \left\{ \int D\xi_2 D\xi_1 + a_1^2 \int \xi_1\xi_2 + \frac{1}{Q_2}\left[\int DJ_1 DJ_2 + a_2^2 \int J_1 J_2\right] + \frac{1}{N_1} \int \xi_1\xi_2 \right\} = 0 \tag{3.4.16}$$

By subtracting equation (3.4.16) from equation (3.4.15) we obtain

$$\left[2(a_1^2 - a_2^2) + Q_1 - Q_2 + \frac{1}{N_1} - \frac{1}{N_2}\right] \int Dw_2 Dw_1 + \left[(a_1^4 - a_2^4) + \frac{a_1^2}{N_1} - \frac{a_2^2}{N_2} \right] \int w_2 w_1$$
$$- \frac{1}{\sqrt{R_1 R_2}} \left[(a_1^2 R_1 - a_2^2 R_2) \int D\theta_1 D\theta_2 + a_1^2 a_2^2 (R_1 - R_2) \int \theta_1\theta_2\right] + \frac{T_1 - T_2}{\sqrt{T_1 T_2}} \int D\xi_1 D\xi_2$$
$$+ \frac{1}{\sqrt{T_1 T_2}} \left[(a_2^2 T_1 - a_1^2 T_2) + \frac{T_1}{N_2} - \frac{T_2}{N_1}\right] \int \xi_1\xi_2 + \frac{1}{\sqrt{T_1 T_2}} \left(\frac{T_1}{Q_1} - \frac{T_2}{Q_2}\right) \int DJ_2 DJ_1$$

$$+\frac{1}{\sqrt{T_1T_2}}\left(\frac{a_1^2T_1}{Q_1}-\frac{a_2^2T_2}{Q_2}\right)\int J_1J_2=0. \tag{3.4.17}$$

Suppose that

$$w_1 \to w_2,\ \theta_1 \to \theta_2,\ \xi_1 \to \xi_2,\ J_1 \to J_2, Q_1 \to Q_2,$$

$$a_1 \to a_2,\ T_1 \to T_2,\ R_1 \to R_2 \text{ and } N_1 \to N_2.$$

Dividing (3.4.17) by $s_1 - s_2$ and taking the limit as $s_1 \to s_2$. Thus (3.4.17)

become

$$4a\frac{\mathrm{d}a}{\mathrm{d}s}\left[\int(Dw)^2+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2}\int(D\theta)^2-\frac{1}{2}\int\xi^2+\frac{1}{2Q}\int J^2\right]$$
$$+\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}s}\left[\int(D\xi)^2+a^2\int\xi^2+\frac{1}{N}\int\xi^2+\frac{1}{Q}\int(DJ)^2+\frac{a^2}{Q}\int J^2\right]$$
$$+\frac{\mathrm{d}Q}{\mathrm{d}s}\left[\int(Dw)^2-\frac{1}{Q^2}\int(DJ)^2-\frac{a^2}{Q^2}\int J^2\right]-\frac{1}{N^2}\frac{\mathrm{d}N}{\mathrm{d}s}\left[\int(Dw)^2+a^2\int w^2-\int\xi^2\right]$$
$$=\frac{a^2}{R}\frac{\mathrm{d}R}{\mathrm{d}s}\left[\int(D\theta)^2+a^2\int\theta^2\right]$$

(3.4.18)

Since

$$R=R(a,T,Q,N)$$

then

$$\frac{\mathrm{d}R}{\mathrm{d}s}=\frac{\partial R}{\partial a}\frac{\mathrm{d}a}{\mathrm{d}s}+\frac{\partial R}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}s}+\frac{\partial R}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}s}+\frac{\partial R}{\partial N}\frac{\mathrm{d}N}{\mathrm{d}s}.$$

Therefore (3.4.18) becomes

$$4a\frac{\mathrm{d}a}{\mathrm{d}s}\left[\int(Dw)^2+a^2\int w^2+\frac{1}{2N}\int w^2-\frac{1}{2}\int(D\theta)^2-\frac{1}{2}\int\xi^2+\frac{1}{2Q}\int J^2\right]$$
$$+\frac{1}{T}\frac{\mathrm{d}T}{\mathrm{d}s}\left[\int(D\xi)^2+a^2\int\xi^2+\frac{1}{N}\int\xi^2+\frac{1}{Q}\int(DJ)^2+\frac{a^2}{Q}\int J^2\right]$$
$$+\frac{\mathrm{d}Q}{\mathrm{d}s}\left[\int(Dw)^2-\frac{1}{Q^2}\int(DJ)^2-\frac{a^2}{Q^2}\int J^2\right]-\frac{1}{N^2}\frac{\mathrm{d}N}{\mathrm{d}s}\left[\int(Dw)^2+a^2\int w^2-\int\xi^2\right]$$
$$=\frac{a^2}{R}\left[\frac{\partial R}{\partial a}\frac{\mathrm{d}a}{\mathrm{d}s}+\frac{\partial R}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}s}+\frac{\partial R}{\partial Q}\frac{\mathrm{d}Q}{\mathrm{d}s}+\frac{\partial R}{\partial N}\frac{\mathrm{d}N}{\mathrm{d}s}\right]\left[\int(D\theta)^2+a^2\int\theta^2\right]$$

(3.4.19)

From (3.4.19), we can show that

$$\frac{\partial R}{\partial a}=\frac{4aR}{\gamma}\left[\int(Dw)^2+(a^2+\frac{1}{2N})\int w^2-\frac{1}{2}\int(D\theta)^2-\frac{1}{2}\int\xi^2+\frac{1}{2Q}\int J^2\right]$$

where $\gamma=a^2\left[\int(D\theta)^2+a^2\int\theta^2\right]$.

Clearly the minimum Rayleigh number occurs when

$$\int(Dw)^2+(a^2+\frac{1}{2N})\int w^2+\frac{1}{2Q}\int J^2=\frac{1}{2}\int(D\theta)^2+\frac{1}{2}\int\xi^2.$$

Also from (3.4.19) we can show that

$$\frac{\partial R}{\partial T}=\frac{R}{T\gamma Q}\left[Q\int(D\xi)^2+a^2Q\int\xi^2+\frac{Q}{N}\int\xi^2+\int(DJ)^2+a^2\int J^2\right]>0.$$

Thus the Rayleigh number R is an increasing function of the Taylor number T. In fact when $Q\to\infty$, $\frac{\partial R}{\partial T}\to 0$ so for higher values of Q the Rayleigh number R does not depend on T.

Moreover from (3.4.19)

$$\frac{\partial R}{\partial Q}=\frac{R}{\gamma}\left[\int(Dw)^2-\frac{1}{Q^2}\int(DJ)^2-\frac{a^2}{Q^2}\int J^2\right].$$

Thus the Rayleigh number is an increasing function of Q provided

$$Q^2\int(Dw)^2 > \int(DJ)^2+a^2\int J^2.$$

It can be shown also from (3.4.19) that

$$\frac{\partial R}{\partial N}=-\frac{R}{\gamma N^2}\left[\int(Dw)^2+a^2\int w^2-\xi^2\right].$$

Which indicates that the Rayleigh number is a decreasing function of the porous medium permeability provided

$$\int(Dw)^2+a^2\int w^2 > \xi^2.$$

The free boundary problem

In the following analysis we shall consider both boundaries to be free but later on we shall present results for the corresponding rigid boundary value problems. For the free boundary value problems

$$w=D^2w=0 \qquad \text{on} \qquad z=0,1.$$

Thus equation (3.3.6) has eigenfunctions $w=A\sin(l\pi z)$ where A is a constant and l is an integer. Consequently $Lw=-\lambda w$ where $\lambda=l^2\pi^2+a^2$ and σ satisfies the fifth order equation,

$$\sigma^5 P_m P_r + \sigma^4\left\{ P_m[P_m + 2P_r(1+P_m)]\lambda + \frac{2P_rP_m^2}{N} \right\} + \sigma^3\left\{ [P_r(P_m+1)^2 + 2P_m(P_m+P_r+1)]\lambda^2 + \frac{2P_m}{N}[P_m + P_r(2+P_m)]\lambda + P_mP_r(2Ql^2\pi^2 + \frac{P_m}{N^2}) + a^2RHP_m^2\lambda^{-1} + TP_rP_m^2l^2\pi^2\lambda^{-1} \right\} + \sigma^2\left\{ [(P_m+1)^2 + 2(P_m+P_r+P_mP_r)]\lambda^3 + \frac{1}{N}[4P_m(1+P_r) + 2(P_r+P_m^2)]\lambda^2 + [\frac{P_m^2}{N^2}(1+\frac{2P_r}{P_m}) + 2Ql^2\pi^2(P_m+P_r+P_mP_r)]\lambda + P_mP_r[(\frac{2Q}{N} + T(2+\frac{P_m}{P_r}))l^2\pi^2 + a^2R\frac{H}{P_r}(2+P_m)] + a^2\frac{R}{N}HP_m^2\lambda^{-1} \right\}$$

$$+ \sigma\left\{ (2P_m+P_r+2)\lambda^4 + \frac{2}{N}(P_r+2P_m+1)\lambda^3 + [2Ql^2\pi^2(1+P_m+P_r) + \frac{1}{N^2}(2P_m+P_r)]\lambda^2 + [a^2RH(1+2P_m) + (\frac{2Q}{N}(P_m+P_r) + T(P_r+2P_m))l^2\pi^2]\lambda + (2a^2\frac{R}{N}P_mH + Q^2l^4\pi^4P_r) + a^2RQHP_ml^2\pi^2\lambda^{-1} \right\} + \lambda^5 + \frac{2}{N}\lambda^4 + (2Ql^2\pi^2 + \frac{1}{N^2})\lambda^3 + [a^2RH + (2\frac{Q}{N} + T)l^2\pi^2]\lambda^2 + (a^2\frac{R}{N}H + Q^2l^4\pi^4)\lambda + a^2RQHl^2\pi^2 = 0. \tag{3.4.20}$$

Since the coefficients of this polynomial are real, then its solutions are one of the following :

1. All solutions are real.
2. Three solutions are real and two are complex conjugate pair solutions.
3. One solution is real and four are complex conjugate solutions.

The stationary instability happens if any real solution is positive while overstability happens if any real part of the complex conjugate solutions is positive. Solutions of (3.4.20) are functions of P_r, P_m, N, Q, T and R and we have to examine how the nature of these solutions depends on P_r, P_m, N, Q, T and R in the context of heating the fluid layer from below.

Stationary convection case

To study the effect of magnetic field, permeability of porous medium and rotation on R for the free boundary problem we set $\sigma = 0$ in equation (3.4.20). Thus we deduce that

$$R = \frac{\lambda}{a^2 C}\left[C^2 + T\lambda \pi^2 l^2\right] \tag{3.4.21}$$

where $C = \lambda^2 + \frac{\lambda}{N} + l^2\pi^2 Q$.

From equation (3.4.21) we fined that

$$\begin{aligned} \frac{\mathrm{d}R}{\mathrm{d}Q} &= \frac{\lambda l^2 \pi^2}{a^2}\left[1 - \frac{T\lambda l^2\pi^2}{C^2}\right] \\ \frac{\mathrm{d}R}{\mathrm{d}N} &= -\frac{\lambda^2}{a^2 N^2}\left[1 - \frac{T l^2 \pi^2 \lambda}{C^2}\right] \\ \frac{\mathrm{d}R}{\mathrm{d}T} &= \frac{\lambda^2 l^2 \pi^2}{a^2 C^2}. \end{aligned} \tag{3.4.22}$$

It is clear from equation $(3.4.22)_1$ that the magnetic field has a stabilizing effect on the system in the absence of rotation. Also it has a stabilizing effect on the system in the presence of rotation provided that

$$T < \frac{C^2}{\lambda l^2 \pi^2} .$$

From equation $(3.4.22)_2$ we find that the permeability of porous medium has a destabilizing effect on the system in the absence of rotation. Also it has a destabilizing effect on the system in the presence of rotation provided that

$$T < \frac{C^2}{l^2 \pi^2 \lambda} .$$

From equation $(3.4.22)_3$ it is clear that the rotation has a stabilizing effect on the system.

The overstability case

Since the equations are so complicated, we were unable to obtain analytical solution for the overstability case but we have produced numerical solutions for the corresponding problem.

3.5 Numerical discussion

The eigenvalue problem (3.3.4) together with the boundary conditions are solved using expansion of Chebyshev polynomials. The relation between the critical Rayliegh number R and the magnetic parameter Q for the stationary convection case is displayed in figures (2)-(6) when both boundaries are free and in figures (14)-(18) when both boundaries are rigid for different values of the porous medium permeability and different values of the Taylor number T. The figures show that as Q increases R increases which means that the magnetic field has a stabilizing effect. Also it appears from the figures that as the permeability of the porous medium, N, decreases R increases which means that as the fluid becomes less porous it becomes more stable. The numerical results are listed in tables (1)-(5) for free boundary conditions and (13)-(17) for rigid boundary conditions. Moreover the presence of porosity delays the conflict which happen between the magnetic field and the rotation in absence of porosity which Chandrasekhar indicated in his book (1961). Similar results were obtained for the overstability case which are displayed in figures (7)-(11) for free boundary conditions and in figures (19)-(23) for rigid boundary conditions and the corresponding numerical results are listed in tables (6)-(10) and (18)-(22) respectively. From figures (19)-(23) it appears that the critical value of the magnetic parameter increases as the permeability of the porous medium decreases.

A comparison between the cases of overstability and stationary convection for the free boundary conditions is displayed in figures (12) and (13). A similar comparison for rigid boundary conditions is displayed in figure (24). The numerical results for free boundary conditions are listed in tables (11), (12) and in table (23) for rigid boundary conditions. The Fortran codes for the free and rigid boundary problems are listed in appendices (II) and (III) respectively.

Table 1. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 0$.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.002)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***	***a***	***R***
1000	5.586	15752.579	4.959	20324.061	4.022	37952.226	3.689	58484.228
2000	6.357	28331.323	5.701	33597.600	4.549	53103.258	4.075	74799.674
3000	6.851	40427.816	6.191	46215.254	4.931	67300.302	4.373	90120.695
4000	7.222	52261.827	6.563	58476.912	5.236	80935.215	4.619	104805.274
5000	7.522	63922.297	6.867	70505.034	5.492	94187.951	4.829	119037.357
6000	7.775	75456.215	7.124	82364.213	5.714	107158.969	5.015	132926.347
7000	7.994	86892.145	7.348	94093.549	5.910	119911.133	5.180	146543.578
8000	8.189	98249.057	7.547	105718.886	6.087	132486.889	5.331	159938.633
9000	8.364	109540.345	7.726	117258.379	6.248	144916.491	5.470	173147.624
10000	8.524	120775.903	7.890	128725.374	6.397	157222.410	5.598	186197.825
11000	8.670	131963.296	8.041	140130.030	6.534	169421.880	5.717	199110.432
12000	8.806	143108.469	8.180	151480.297	6.663	181528.458	5.829	211902.304
13000	8.933	154216.203	8.310	162782.537	6.783	193553.043	5.935	224587.113
14000	9.051	165290.409	8.433	174041.942	6.897	205504.550	6.035	237176.130
15000	9.163	176334.343	8.548	185262.813	7.004	217390.382	6.130	249678.775
20000	9.643	231183.329	9.042	240898.704	7.471	276023.451	6.545	311139.227
25000	10.030	285561.006	9.441	295944.719	7.854	333659.490	6.890	371273.086
30000	10.357	339593.703	9.778	350563.897	8.181	390579.992	7.186	430449.509
35000	10.641	393358.478	10.071	404854.606	8.467	446954.283	7.447	488892.687
40000	10.893	446906.478	10.331	458881.773	8.722	502893.658	7.681	546750.890
45000	11.119	500274.556	10.564	512691.008	8.953	558475.930	7.894	604128.227
50000	11.326	553489.076	10.777	566315.850	9.164	613758.052	8.089	661101.128

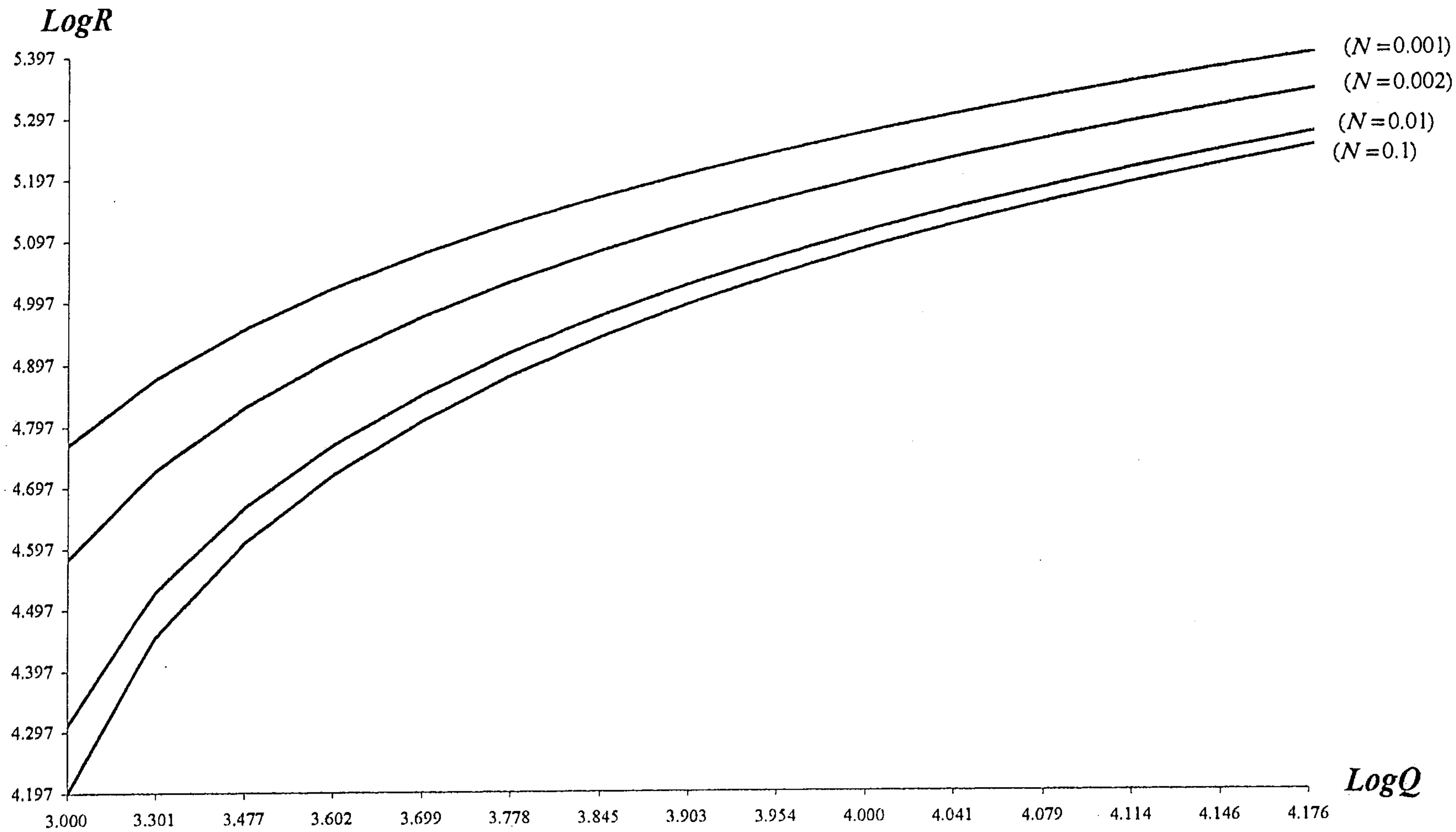

Figure 2. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 0$.

Table 2. The relation between R and Q for the stationary convection case when both boundaries are free for T=1000.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	5.582	15797.159	4.958	20356.922	4.022	37969.786	3.689	58496.020
2000	6.355	28358.437	5.699	33618.752	4.549	53115.649	4.075	74808.553
3000	6.849	40448.128	6.190	46231.528	4.931	67310.211	4.373	90127.994
4000	7.221	52278.392	6.562	58490.411	5.236	80943.625	4.619	104811.560
5000	7.521	63936.447	6.866	70516.707	5.492	94195.341	4.829	119042.931
6000	7.774	75468.660	7.123	82374.000	5.714	107165.610	5.015	132931.387
7000	7.993	86903.312	7.347	94102.920	5.910	119917.199	5.180	146548.203
8000	8.188	98259.226	7.546	105727.474	6.087	132492.494	5.331	159942.921
9000	8.363	109549.710	7.726	117266.331	6.248	144921.719	5.470	173151.635
10000	8.523	120784.603	7.890	128732.796	6.397	157227.322	5.598	186201.602
11000	8.670	131971.436	8.040	140137.003	6.534	169426.521	5.717	199114.008
12000	8.806	143116.130	8.180	151486.884	6.663	181532.866	5.829	211905.705
13000	8.932	154223.448	8.310	162788.788	6.783	193557.246	5.935	224590.360
14000	9.051	165297.290	8.432	174047.898	6.897	205508.571	6.035	237179.241
15000	9.163	176340.902	8.547	185268.505	7.004	217394.242	6.130	249681.765
20000	9.642	231188.701	9.041	240903.419	7.471	276026.704	6.545	311141.758
25000	10.030	285565.610	9.441	295948.792	7.854	333662.338	6.890	371275.310
30000	10.357	339597.762	9.778	350567.512	8.181	390582.547	7.188	430451.510
35000	10.641	393362.128	10.071	404857.873	8.467	446956.614	7.447	488894.517
40000	10.893	446909.983	10.330	458884.766	8.722	502895.811	7.681	546752.584
45000	11.119	500277.627	10.564	512693.778	8.953	558477.937	7.894	604129.809
50000	11.325	553491.933	10.777	566318.435	9.164	613759.937	8.089	661102.617

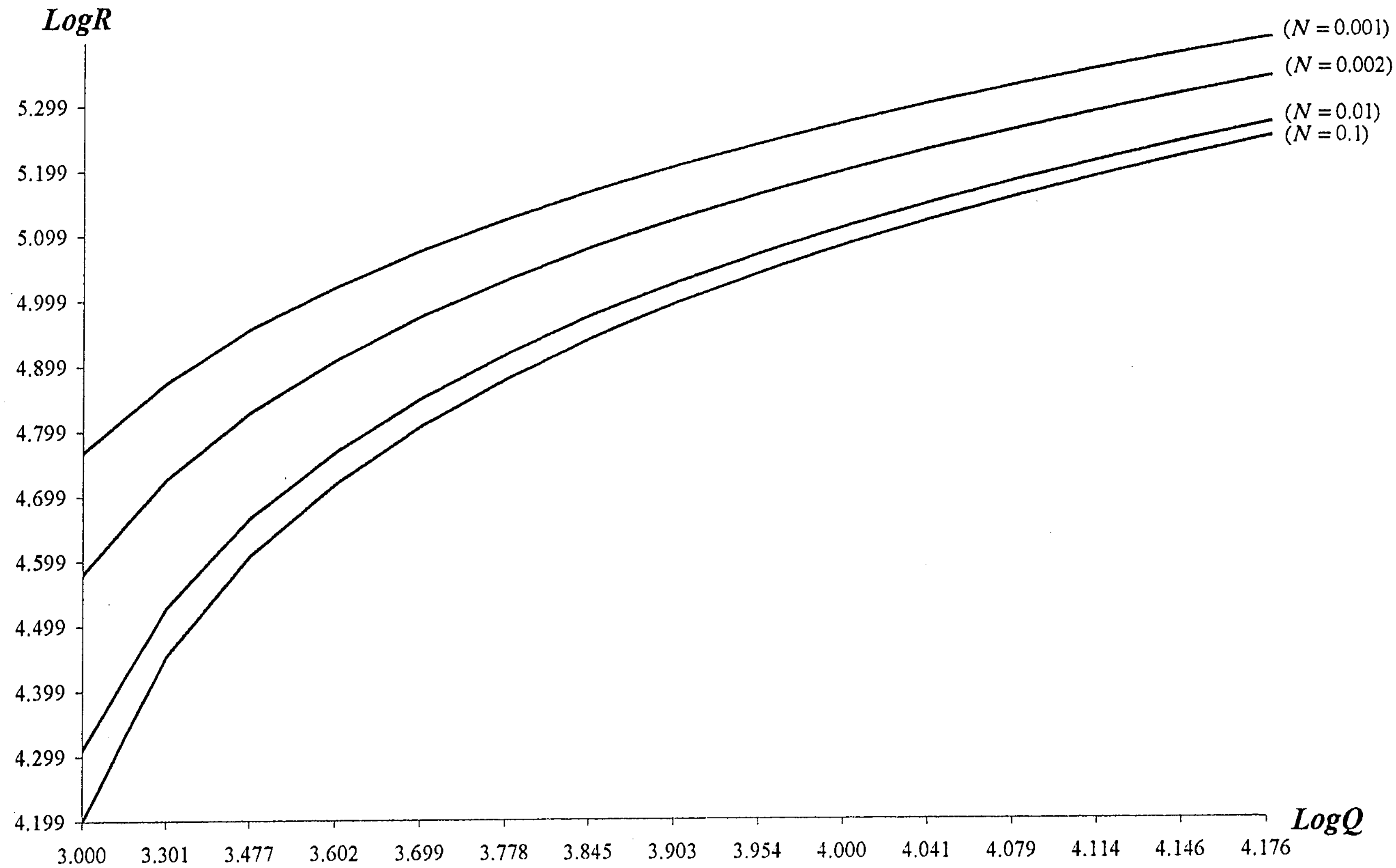

Figure 3. The relation between R and Q for the stationary convection case when both boundaries are free for T=1000.

Table 3. The relation between R and Q for the stationary convection case when both boundaries are free for $T=10000$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	5.545	16197.533	4.947	20652.581	4.025	38127.808	3.691	58602.138
2000	6.330	28602.030	5.688	33808.996	4.549	53227.167	4.076	74888.469
3000	6.832	40630.683	6.180	46377.906	4.930	67399.389	4.373	90193.679
4000	7.207	52427.311	6.554	58611.832	5.235	81019.308	4.619	104868.135
5000	7.509	64063.677	6.859	70621.703	5.491	94261.838	4.829	119093.097
6000	7.764	75580.575	7.117	82467.801	5.712	107225.379	5.014	132976.758
7000	7.985	87003.751	7.342	94187.221	5.909	119971.785	5.180	146589.826
8000	8.181	98350.696	7.542	105804.734	6.086	132542.942	5.331	159981.521
9000	8.357	109633.946	7.722	117337.867	6.247	144968.769	5.469	173187.733
10000	8.517	120862.863	7.886	128799.571	6.396	157271.523	5.597	186235.590
11000	8.664	132044.662	8.037	140199.744	6.533	169468.292	5.717	199146.187
12000	8.801	143185.049	8.176	151547.153	6.662	181572.532	5.829	211936.313
13000	8.928	154288.632	8.307	162845.034	6.782	193595.070	5.934	224619.589
14000	9.047	165359.200	8.429	174101.481	6.896	205544.765	6.034	237207.245
15000	9.159	176399.914	8.545	185319.723	7.003	217428.980	6.129	249708.674
20000	9.640	231237.041	9.039	240945.843	7.470	276055.977	6.545	311164.537
25000	10.028	285607.038	9.439	295985.447	7.853	333687.970	6.889	371295.324
30000	10.355	339634.293	9.777	350600.037	8.180	390605.542	7.186	430469.518
35000	10.639	393394.979	10.070	404887.271	8.467	446977.593	7.447	488910.987
40000	10.891	446939.951	10.329	458911.699	8.722	502915.185	7.681	546767.829
45000	11.118	500305.265	10.563	512718.708	8.953	558495.999	7.894	604144.050
50000	11.324	553517.643	10.776	566341.700	9.164	613776.899	8.089	661116.017

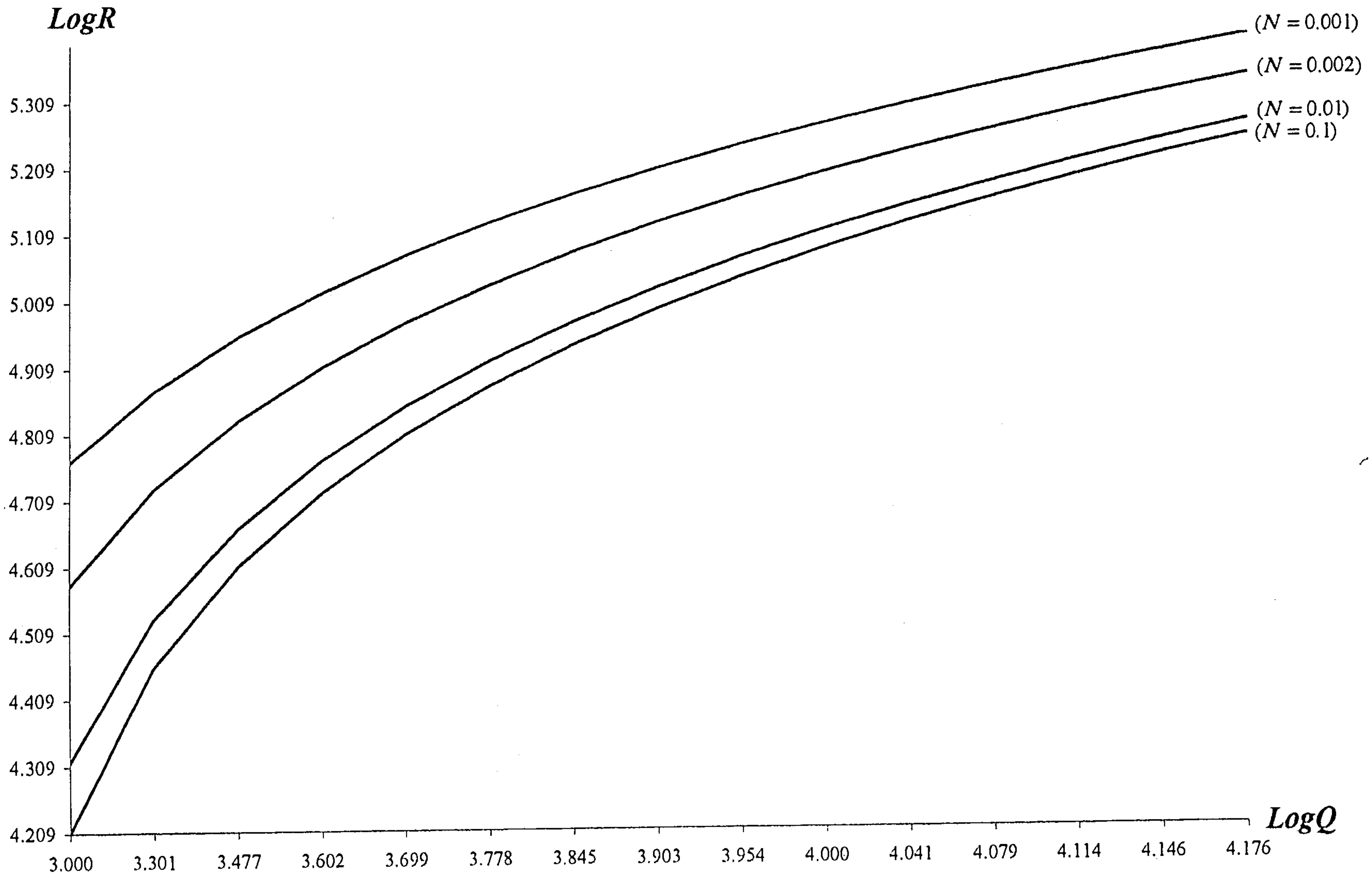

Figure 4. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 10000$.

Table 4. The relation between R and Q for the stationary convection case when both boundaries are free for T=50000.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	5.383	17959.025	4.903	21964.578	4.035	38829.831	3.699	59073.456
2000	6.223	29675.402	5.640	34651.866	4.547	53722.795	4.078	75243.620
3000	6.754	41436.617	6.139	47026.432	4.926	67795.675	4.374	90485.611
4000	7.146	53085.606	6.520	59149.935	5.229	81355.588	4.618	105119.579
5000	7.460	64626.612	6.829	71087.154	5.485	94557.286	4.828	119316.051
6000	7.723	76076.081	7.091	82881.177	5.707	107490.920	5.013	133178.393
7000	7.949	87448.671	7.319	94561.109	5.904	120214.296	5.179	146774.805
8000	8.149	98756.043	7.521	106147.458	6.081	132767.064	5.329	160153.060
9000	8.329	110007.359	7.703	117655.253	6.242	145177.799	5.468	173348.156
10000	8.492	121209.875	7.868	129095.875	6.391	157467.897	5.596	186386.635
11000	8.641	132369.423	8.020	140478.177	6.528	169653.867	5.715	199289.194
12000	8.780	143490.761	8.162	151809.211	6.657	181748.761	5.827	212072.336
13000	8.908	154577.823	8.293	163094.693	6.778	193763.113	5.933	224749.477
14000	9.029	165633.901	8.416	174339.342	6.891	205705.566	6.033	237331.692
15000	9.142	176661.787	8.532	185547.099	6.999	217583.318	6.128	249828.255
20000	9.627	231451.643	9.030	241134.233	7.467	276186.038	6.543	311265.764
25000	10.018	285791.001	9.431	296148.242	7.850	333801.858	6.888	371384.268
30000	10.347	339796.536	9.770	350744.512	8.178	390707.719	7.184	430549.542
35000	10.632	393540.893	10.064	405017.868	8.464	447070.810	7.446	488984.177
40000	10.885	447073.071	10.325	459031.352	8.720	503001.277	7.680	546835.577
45000	11.113	500428.044	10.559	512829.468	8.951	558576.258	7.893	604207.339
50000	11.320	553631.862	10.772	566445.063	9.162	613852.275	8.088	661175.568

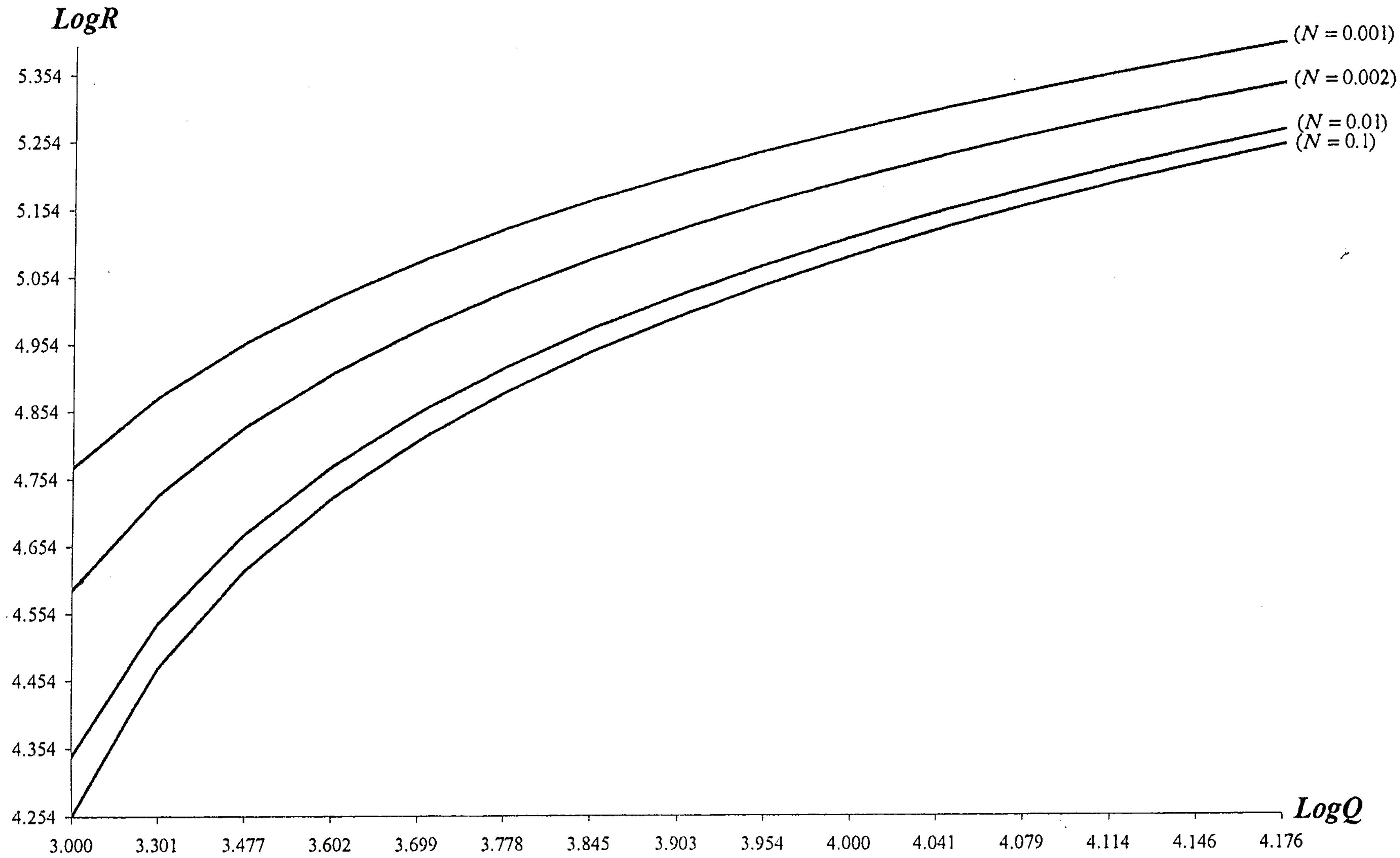

Figure 5. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 50000$.

Table 5. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 100000$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	5.195	20121.610	4.850	23600.078	4.047	39706.690	3.709	59661.886
2000	6.094	30996.355	5.581	35699.480	4.545	54342.316	4.081	75687.491
3000	6.659	42431.786	6.089	47832.476	4.920	68290.900	4.374	90850.526
4000	7.072	53900.395	6.477	59819.061	5.223	81775.738	4.617	105433.879
5000	7.399	65324.533	6.792	71666.239	5.478	94926.375	4.827	119594.726
6000	7.671	76691.154	7.058	83395.715	5.700	107822.624	5.011	133430.413
7000	7.905	88001.461	7.290	95026.684	5.897	120517.222	5.177	147005.998
8000	8.110	99260.032	7.495	106574.375	6.074	133047.015	5.327	160367.450
9000	8.294	110471.909	7.679	118050.725	6.236	145438.898	5.465	173548.648
10000	8.461	121641.785	7.847	129465.169	6.385	157713.188	5.594	186575.405
11000	8.613	132773.798	8.000	140825.277	6.523	169885.672	5.713	199467.916
12000	8.754	143871.544	8.143	152137.204	6.651	181968.895	5.825	212242.330
13000	8.884	154938.128	8.276	163406.035	6.772	193973.025	5.931	224911.803
14000	9.006	165976.238	8.400	174636.014	6.886	205906.435	6.031	237487.217
15000	9.121	176988.207	8.517	185830.731	6.994	217776.116	6.126	249977.698
20000	9.612	231719.342	9.018	241369.348	7.463	276348.523	6.541	311392.271
25000	10.005	286020.583	9.422	296351.478	7.847	333944.147	6.886	371495.423
30000	10.337	339999.072	9.762	350924.916	8.174	390835.382	7.183	430649.552
35000	10.624	393723.085	10.057	405180.968	8.461	447187.285	7.444	489075.647
40000	10.878	447239.313	10.319	459180.801	8.717	503108.854	7.679	546920.247
45000	11.106	500581.392	10.553	512967.822	8.948	558676.549	7.891	604286.436
50000	11.314	553774.532	10.767	566574.188	9.160	613946.466	8.087	661249.996

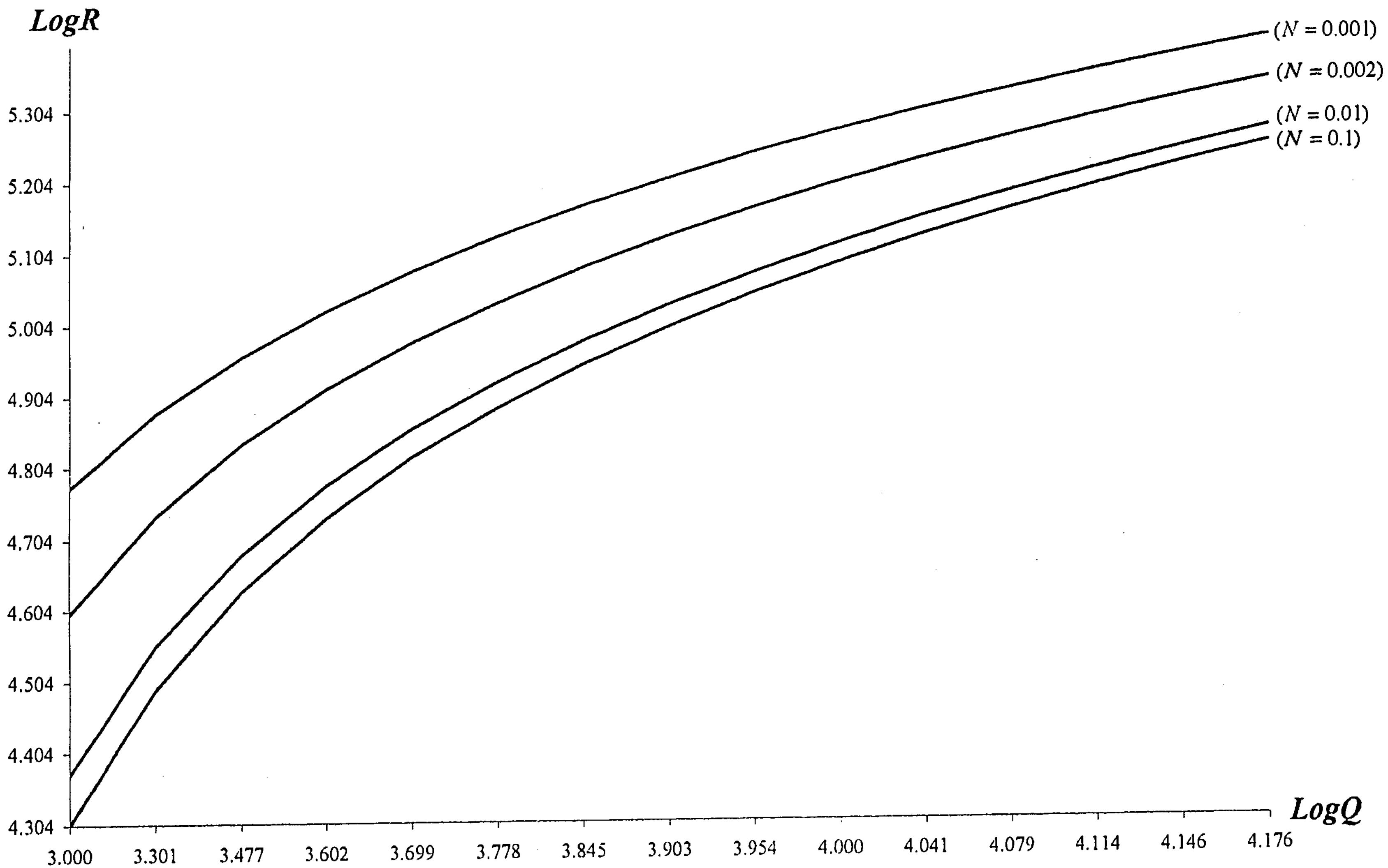

Figure 6. The relation between R and Q for the stationary convection case when both boundaries are free for $T = 100000$.

Table 6. The relation between R and Q for overstability case when both boundaries are free for $T=0$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	3.790	4927.882	3.650	10294.619	3.330	30583.246	3.246	55369.245
2000	4.309	7627.104	4.098	13868.419	3.538	34978.334	3.371	59989.789
3000	4.653	10122.290	4.427	17160.801	3.715	39159.502	3.484	64458.917
4000	4.914	12503.921	4.691	20280.011	3.870	43181.301	3.586	68803.747
5000	5.127	14809.928	4.914	23277.813	4.008	47078.211	3.681	73044.116
6000	5.307	17060.636	5.108	26183.918	4.133	50873.688	3.768	77195.056
7000	5.465	19268.491	5.281	29017.324	4.248	54584.556	3.850	81268.282
8000	5.604	21441.785	5.436	31791.091	4.355	58223.372	3.927	85273.135
9000	5.730	23586.391	5.578	34514.673	4.454	61799.809	4.000	89217.209
10000	5.845	25706.648	5.709	37195.192	4.547	65321.509	4.069	93106.774
11000	5.951	27805.877	5.831	39838.176	4.635	68794.636	4.135	96947.081
12000	6.048	29886.691	5.944	42448.025	4.718	72224.249	4.197	100742.579
13000	6.140	31951.186	6.051	45028.313	4.797	75614.563	4.257	104497.077
14000	6.225	34001.082	6.152	47581.990	4.873	78969.134	4.315	108213.868
15000	6.306	36037.807	6.247	50111.531	4.945	82290.995	4.371	111895.818
20000	6.653	46060.437	6.662	62461.317	5.268	98490.327	4.621	129861.676
25000	6.933	55878.899	7.003	74428.651	5.544	114153.909	4.837	147236.994
30000	7.169	65548.408	7.295	86113.246	5.786	129414.110	5.028	164160.729
35000	7.375	75102.556	7.550	97576.734	6.003	144354.646	5.200	180723.838
40000	7.557	84563.660	7.777	108860.551	6.201	159032.605	5.358	196989.761
45000	7.721	93947.485	7.983	119994.235	6.384	173489.077	5.503	213004.870
50000	7.870	103265.677	8.171	130999.772	6.554	187754.864	5.638	228804.297

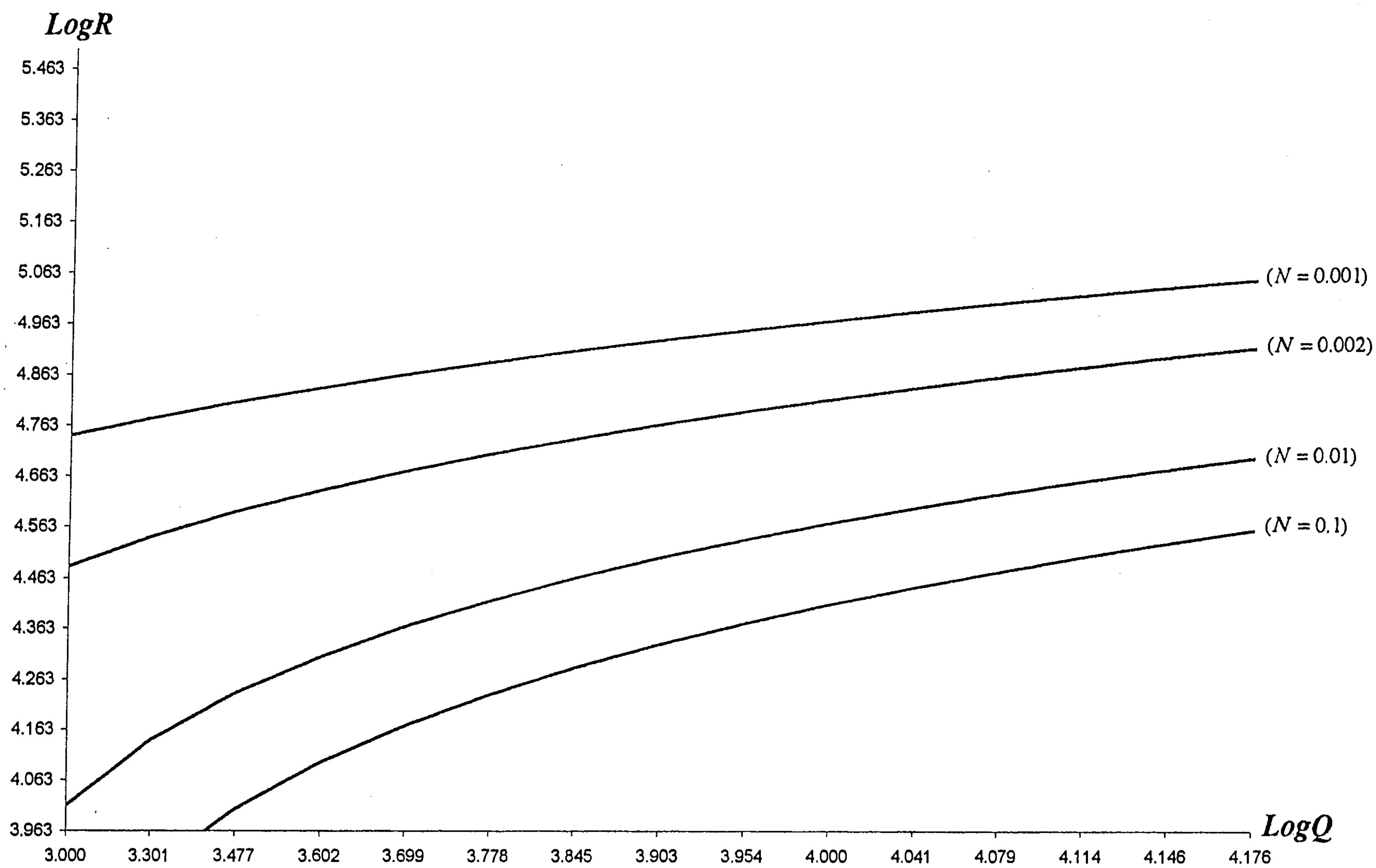

Figure 7. The relation between R and Q for the overstability case when both boundaries are free for $T = 0$.

Table 7. The relation between R and Q for the overstability case when both boundaries are free for $T=1000$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	3.772	5092.313	3.650	10374.345	3.331	30617.360	3.247	55389.417
2000	4.288	7727.543	4.091	13914.628	3.539	35004.272	3.372	60006.888
3000	4.635	10197.027	4.419	17191.886	3.715	39180.051	3.484	64473.662
4000	4.899	12564.392	4.683	20302.524	3.870	43198.015	3.586	68816.627
5000	5.114	14861.194	4.907	23294.840	4.008	47092.050	3.681	73055.479
6000	5.296	17105.414	5.101	26197.153	4.133	50885.287	3.768	77205.158
7000	5.455	19308.419	5.224	29027.796	4.248	54594.357	3.850	81277.320
8000	5.595	21477.936	5.430	31799.472	4.354	58231.698	3.927	85281.261
9000	5.722	23619.506	5.572	34521.425	4.453	61806.902	4.000	89224.544
10000	5.837	25737.263	5.704	37200.644	4.546	65327.555	4.069	93113.417
11000	5.944	27834.394	5.825	39842.571	4.634	68799.782	4.135	96953.111
12000	6.042	29913.418	5.939	42451.547	4.717	72228.613	4.197	100748.064
13000	6.134	31976.367	6.046	45031.104	4.797	75618.241	4.257	104502.072
14000	6.220	34024.912	6.147	47584.164	4.872	78972.205	4.315	108218.420
15000	6.301	36060.444	6.243	50113.178	4.945	82293.526	4.371	111895.818
20000	6.648	46078.716	6.659	62461.205	5.268	98490.855	4.621	129861.676
25000	6.929	55894.391	7.000	74427.589	5.543	114153.149	4.837	147236.994
30000	7.166	65561.945	7.292	86111.622	5.785	129412.459	5.028	164160.729
35000	7.372	75114.637	7.547	97574.759	6.003	144352.346	5.200	180723.838
40000	7.555	84574.610	7.775	108858.350	6.201	159029.817	5.357	196989.761
45000	7.719	93957.526	7.981	119991.886	6.384	173485.911	5.503	213004.870
50000	7.868	103274.971	8.169	130997.327	6.554	187751.400	5.638	228804.297

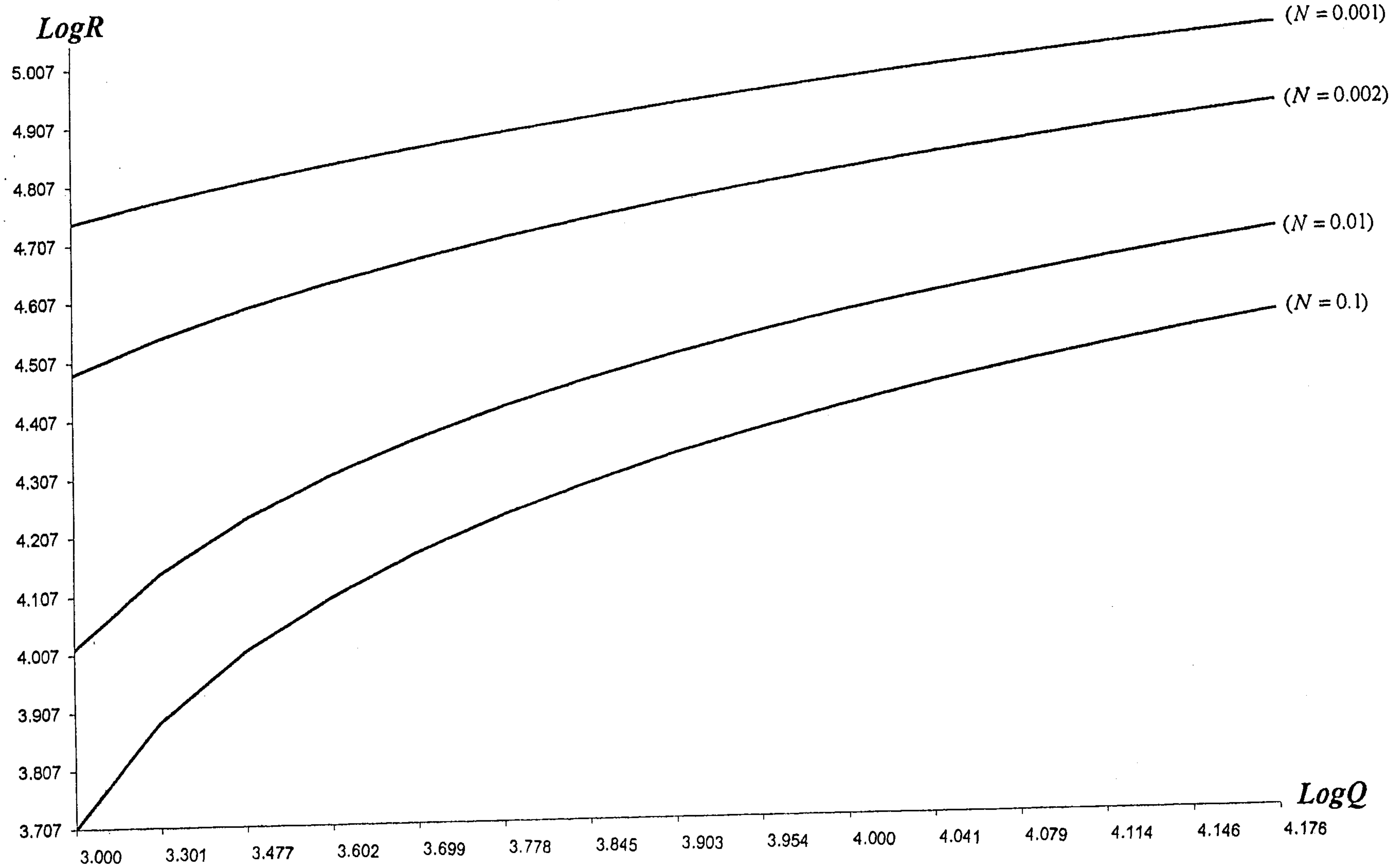

Figure 8. The relation between R and Q for the overstability case when both boundaries are free for $T = 1000$.

Table 8. The relation between R and Q for the overstability case when both boundaries are free for $T=10000$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***	***a***	***R***
1000	3.543	6004.873	3.642	11058.716	3.342	30923.592	3.251	55570.814
2000	4.054	8266.378	4.036	14309.599	3.543	35237.296	3.374	60160.683
3000	4.436	10607.754	4.354	17456.335	3.716	39364.729	3.486	64606.300
4000	4.732	12908.907	4.618	20493.151	3.868	43348.263	3.587	68932.446
5000	4.971	15163.902	4.844	23438.270	4.005	47216.468	3.681	73157.707
6000	5.172	17378.525	5.041	26308.001	4.129	50989.575	3.768	77296.056
7000	5.345	19559.023	5.217	29114.930	4.243	54682.496	3.850	81358.643
8000	5.498	21710.591	5.376	31868.668	4.349	58306.580	3.927	85354.381
9000	5.634	23837.359	5.521	34576.647	4.448	61870.699	3.999	89290.550
10000	5.758	25942.608	5.654	37244.725	4.541	65381.944	4.068	93173.189
11000	5.871	28028.970	5.779	39877.609	4.629	68846.081	4.133	97007.374
12000	5.975	30098.585	5.894	42479.114	4.712	72267.881	4.196	100797.418
13000	6.071	32153.215	6.003	45052.432	4.791	75651.341	4.256	104547.025
14000	6.162	34194.333	6.106	47600.220	4.867	78999.851	4.314	108259.401
15000	6.247	36223.182	6.203	50124.737	4.939	82316.315	4.369	111937.346
20000	6.608	46215.836	6.625	62457.811	5.262	98495.633	4.619	129887.676
25000	6.897	56013.983	6.972	74416.158	5.538	114146.343	4.835	147252.322
30000	7.139	65668.631	7.267	86095.478	5.780	129397.630	5.026	164168.246
35000	7.349	75211.348	7.525	97555.706	5.998	144331.682	5.198	180725.379
40000	7.535	84663.340	7.755	108837.448	6.196	159004.754	5.356	196986.581
45000	7.701	94039.703	7.962	119969.795	6.379	173457.448	5.501	212997.860
50000	7.853	103351.652	8.152	130974.479	6.550	187720.254	5.636	228794.121

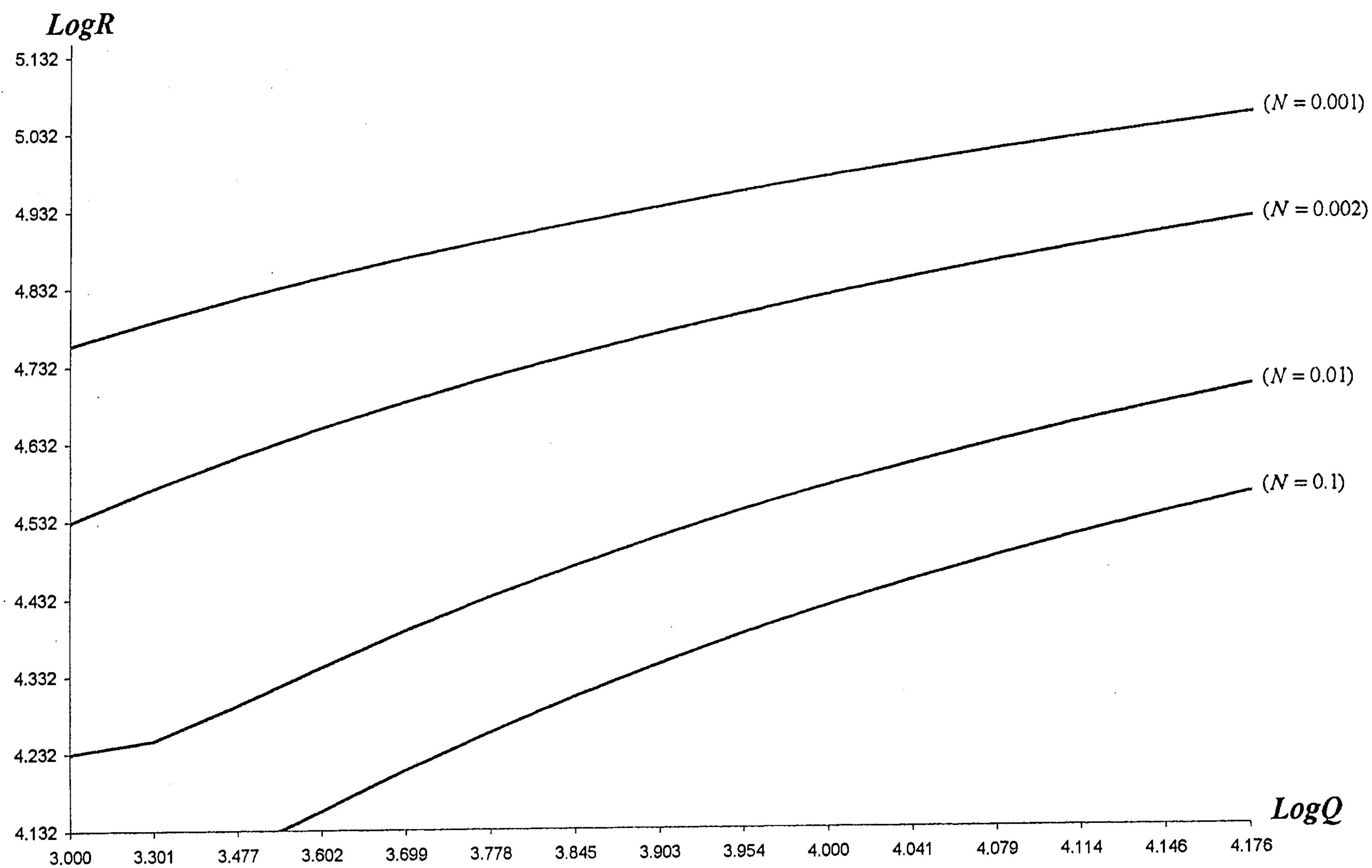

Figure 9. The relation between R and Q for the overstability case when both boundaries are free for T =10000.

Table 9. The relation between R and Q for the overstability case when both boundaries are free for T=50000.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.002)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***	***a***	***R***
1000	3.396	9317.322	3.627	13794.902	3.389	32268.455	3.270	56373.790
2000	3.671	9817.488	3.858	15861.708	3.560	36264.504	3.387	60842.225
3000	3.992	11539.219	4.135	18470.195	3.717	40180.202	3.493	65194.477
4000	4.277	13548.277	4.388	21204.174	3.860	44012.361	3.591	69446.536
5000	4.525	15646.384	4.614	23957.086	3.991	47766.791	3.683	73611.367
6000	4.744	17770.396	4.817	26695.141	4.111	51451.140	3.768	77699.527
7000	4.938	19896.157	4.999	29406.993	4.223	55072.805	3.848	81719.692
8000	5.112	22013.683	5.166	32089.390	4.327	58638.373	3.924	85679.065
9000	5.269	24118.885	5.318	34742.246	4.425	62153.548	3.996	89583.689
10000	5.413	26210.321	5.459	37366.738	4.517	65623.229	4.064	93438.686
11000	5.545	28287.794	5.591	39964.504	4.605	69051.619	4.129	97248.433
12000	5.667	30351.693	5.713	42537.291	4.688	72442.341	4.191	101016.706
13000	5.780	32402.667	5.829	45086.809	4.767	75798.530	4.251	104746.784
14000	5.885	34441.466	5.937	47614.659	4.842	79122.916	4.308	108441.536
15000	5.984	36468.857	6.040	50122.318	4.914	82417.888	4.363	112103.492
20000	6.401	46459.098	6.486	62400.709	5.238	98517.448	4.613	129991.807
25000	6.729	56255.443	6.851	74331.117	5.515	114116.781	4.828	147313.821
30000	6.999	65906.743	7.159	85995.013	5.758	129332.445	5.019	164198.534
35000	7.230	75444.842	7.429	97446.421	5.977	144240.547	5.192	180731.785
40000	7.431	84891.453	7.668	108723.118	6.176	158894.045	5.349	196974.104
45000	7.609	94262.059	7.883	119852.703	6.360	173331.585	5.495	212970.072
50000	7.771	103568.133	8.079	130856.062	6.531	187582.431	5.630	228753.667

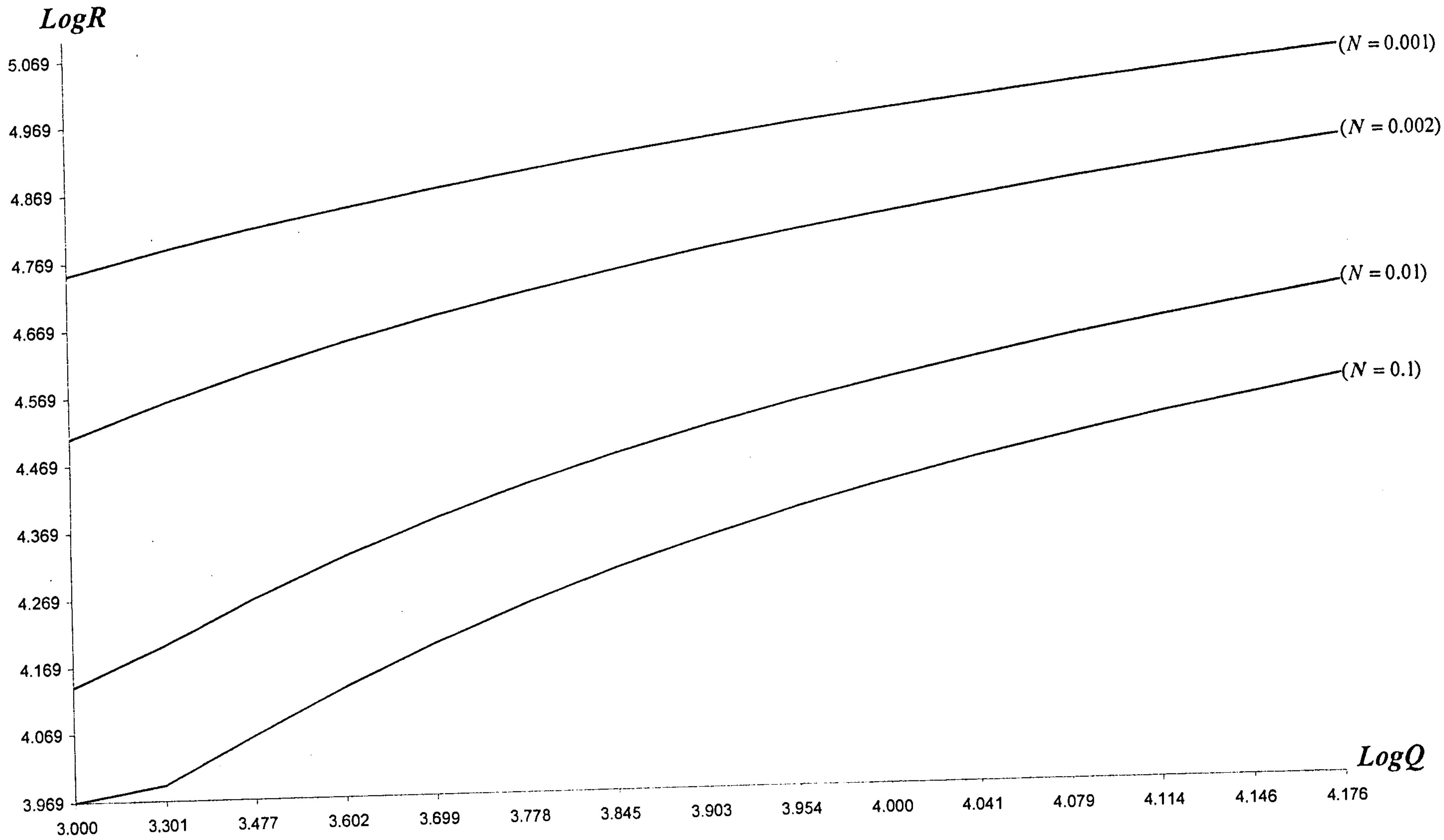

Figure 10. The relation between R and Q for the overstability case when both boundaries are free for T=50000.

Table 10. The relation between R and Q for the overstability case when both boundaries are free for $T=100000$.

	(N=0.1)		(N=0.01)		(N=0.002)		(N=0.001)	
Q	a	R	a	R	a	R	a	R
1000	3.424	13538.330	3.665	17026.519	3.443	33916.590	3.293	57370.344
2000	3.550	11893.484	3.750	17704.169	3.581	37531.207	3.402	61689.722
3000	3.803	12774.614	3.974	19658.976	3.719	41188.748	3.502	65926.737
4000	4.050	14330.805	4.205	22018.737	3.851	44835.135	3.597	70086.995
5000	4.274	16149.911	4.420	24531.723	3.975	48449.492	3.685	74176.911
6000	4.477	18090.161	4.618	27104.570	4.091	52024.376	3.768	78202.719
7000	4.662	20090.420	4.799	29696.985	4.200	55558.065	3.846	82170.134
8000	4.831	22120.534	4.967	32289.935	4.302	59051.328	3.921	86084.264
9000	4.986	24164.320	5.122	34873.994	4.398	62505.983	3.991	89949.628
10000	5.130	26212.614	5.266	37444.452	4.490	65924.238	4.059	93770.210
11000	5.264	28260.061	5.401	39999.061	4.576	69308.374	4.123	97549.525
12000	5.390	30303.490	5.527	42536.904	4.659	72660.599	4.185	101290.680
13000	5.507	32341.034	5.646	45057.802	4.737	75982.987	4.244	104996.429
14000	5.618	34371.625	5.759	47561.987	4.813	79277.457	4.301	108669.225
15000	5.722	36394.704	5.866	50049.918	4.885	82545.762	4.356	112311.255
20000	6.169	46394.666	6.331	62267.164	5.210	98546.203	4.604	130122.272
25000	6.526	56219.086	6.712	74169.539	5.487	114081.525	4.820	147391.118
30000	6.821	65899.389	7.034	85820.203	5.732	129252.699	5.011	164236.889
35000	7.073	75462.930	7.315	97265.759	5.952	144128.324	5.184	180740.328
40000	7.291	84930.683	7.563	108540.551	6.152	158757.276	5.341	196959.064
45000	7.485	94318.485	7.787	119670.435	6.337	173175.782	5.487	212935.903
50000	7.659	103638.423	7.990	130675.351	6.509	187411.579	5.623	228703.667

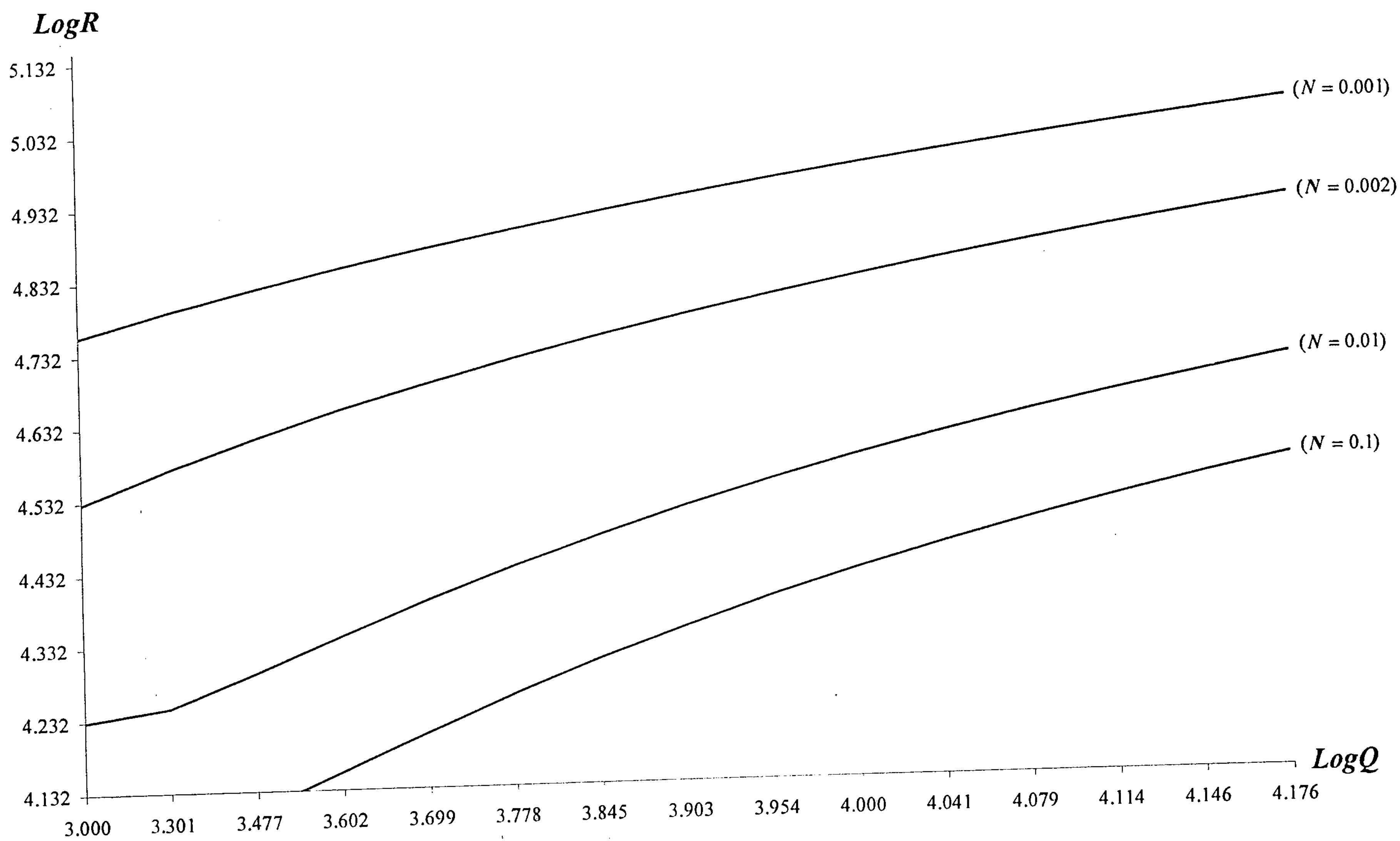

Figure 11. The relation between R and Q for the overstability case when both boundaries are free for T=100000.

Table 11. A comparison between the cases of overstability and stationary convection when both boundaries are free for $T=10000$.

	(N=0.01)				(N=0.001)			
	Overstability		Stationary		Overstability		Stationary	
`Q	a	R	a	R	a	R	a	R
1000	3.642	11058.716	4.947	20652.581	3.251	55570.814	3.691	58602.138
2000	4.036	14309.599	5.688	33808.996	3.374	60160.683	4.076	74888.469
3000	4.354	17456.335	6.180	46377.906	3.486	64606.300	4.373	90193.679
4000	4.618	20493.151	6.554	58611.832	3.587	68932.446	4.619	104868.135
5000	4.844	23438.270	6.859	70621.703	3.681	73157.707	4.829	119093.097
6000	5.041	26308.001	7.117	82467.801	3.768	77296.056	5.014	132976.758
7000	5.217	29114.930	7.342	94187.221	3.850	81358.643	5.180	146589.826
8000	5.376	31868.668	7.542	105804.734	3.927	85354.381	5.331	159981.521
9000	5.521	34576.647	7.722	117337.867	3.999	89290.550	5.469	173187.733
10000	5.654	37244.725	7.886	128799.571	4.068	93173.189	5.597	186235.590
11000	5.779	39877.609	8.037	140199.744	4.133	97007.374	5.717	199146.187
12000	5.894	42479.114	8.176	151547.153	4.196	100797.418	5.829	211936.313
13000	6.003	45052.432	8.307	162845.034	4.256	104547.025	5.934	224619.589
14000	6.106	47600.220	8.429	174101.481	4.314	108259.401	6.034	237207.245
15000	6.203	50124.737	8.545	185319.723	4.369	111937.346	6.129	249708.674
20000	6.625	62457.811	9.039	240945.843	4.619	129887.676	6.545	311164.537
25000	6.972	74416.158	9.439	295985.447	4.835	147252.322	6.889	371295.324
30000	7.267	86095.478	9.777	350600.037	5.026	164168.246	7.186	430469.518
35000	7.525	97555.706	10.070	404887.271	5.198	180725.379	7.447	488910.987
40000	7.755	108837.448	10.329	458911.699	5.356	196986.581	7.681	546767.829

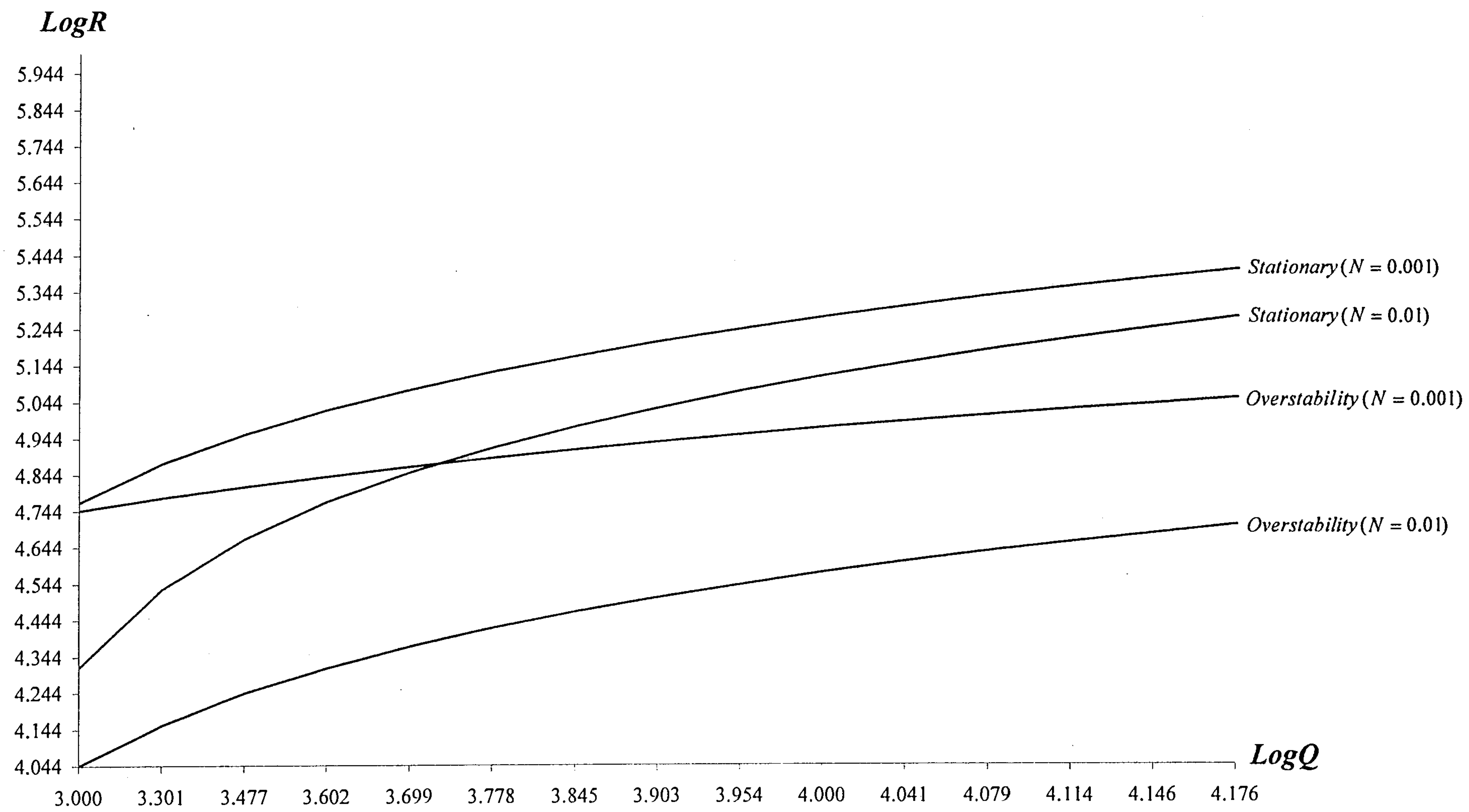

Figure 12. A comparison between the cases of overstability and stationary convection when both boundaries are free for T =10000.

Table 12. A comparison between the cases of overstability and stationary convection when both boundaries are free for T =50000.

	(*N*=0.01)				(*N*=0.001)			
	Overstability		Stationary		Overstability		Stationary	
Q	a	R	a	R	a	R	a	R
1000	3.627	13794.902	4.903	21964.578	3.270	56373.790	3.699	59073.456
2000	3.858	15861.708	5.640	34651.866	3.387	60842.225	4.078	75243.620
3000	4.135	18470.195	6.139	47026.432	3.493	65194.477	4.374	90485.611
4000	4.388	21204.174	6.520	59149.935	3.591	69446.536	4.618	105119.579
5000	4.614	23957.086	6.829	71087.154	3.683	73611.367	4.828	119316.051
6000	4.817	26695.141	7.091	82881.177	3.768	77699.527	5.013	133178.393
7000	4.999	29406.993	7.319	94561.109	3.848	81719.692	5.179	146774.805
8000	5.166	32089.390	7.521	106147.458	3.924	85679.065	5.329	160153.060
9000	5.318	34742.246	7.703	117655.253	3.996	89583.689	5.468	173348.156
10000	5.459	37366.738	7.868	129095.875	4.064	93438.686	5.596	186386.635
11000	5.591	39964.504	8.020	140478.177	4.129	97248.433	5.715	199289.194
12000	5.713	42537.291	8.162	151809.211	4.191	101016.706	5.827	212072.336
13000	5.829	45086.809	8.293	163094.693	4.251	104746.784	5.933	224749.477
14000	5.937	47614.659	8.416	174339.342	4.308	108441.536	6.033	237331.692
15000	6.040	50122.318	8.532	185547.099	4.363	112103.492	6.128	249828.255
20000	6.486	62400.709	9.030	241134.233	4.613	129991.807	6.543	311265.764
25000	6.851	74331.117	9.431	296148.242	4.828	147313.821	6.888	371384.268
30000	7.159	85995.013	9.770	350744.512	5.019	164198.534	7.184	430549.542
35000	7.429	97446.421	10.064	405017.868	5.192	180731.785	7.446	488984.177
40000	7.668	108723.118	10.325	459031.352	5.349	196974.104	7.680	546835.577

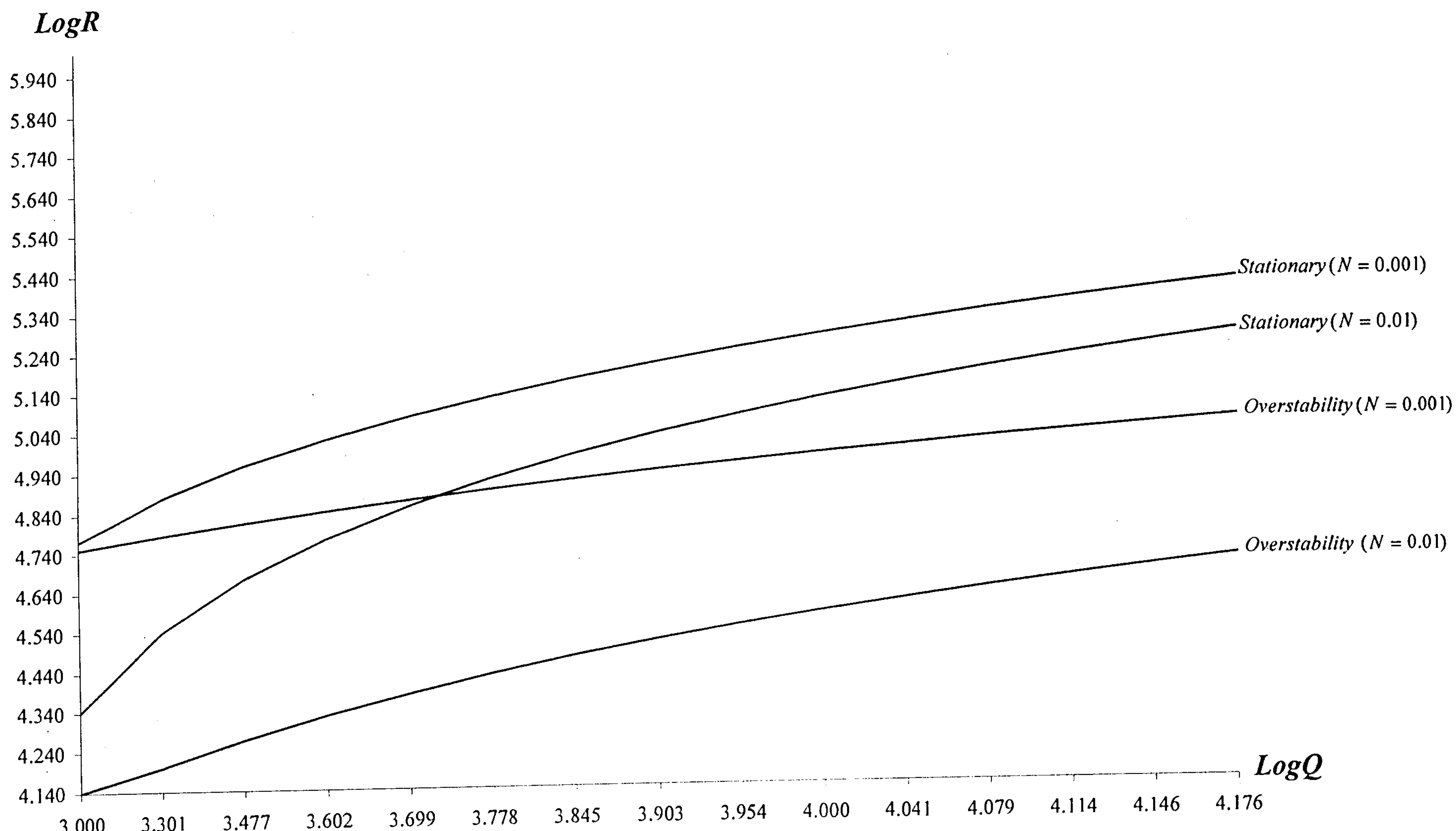

Figure 13. A comparison between the cases of overstability and stationary convection when both boundaries are free for T =50000.

Table 13. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 0$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	3.149	2150.603	3.231	6039.782	3.214	43149.035
400	4.883	9121.076	4.343	13328.622	3.473	50985.413
800	5.499	14914.659	4.881	19508.421	3.733	58322.483
1200	5.904	20367.246	5.267	25279.646	3.835	65290.865
1600	6.584	25717.246	5.563	30817.936	4.013	72007.468
2000	7.303	31155.863	5.807	36200.093	4.151	78525.055
2400	7.335	36034.411	6.016	41467.992	4.275	84880.244
2800	7.708	41206.647	6.200	46647.462	4.389	91100.746
3200	7.109	45739.882	6.364	51755.777	4.494	97207.187
3600	7.163	50613.871	6.512	56805.171	4.593	103215.411
4000	7.310	55454.280	6.648	61804.690	4.669	109138.936
4400	7.432	60260.751	6.773	66764.344	4.770	114984.701
4800	7.543	65037.537	6.890	71680.272	4.892	120770.849
5200	7.649	69788.044	6.999	76566.122	4.930	126482.727
5600	7.750	74515.033	7.101	81422.357	5.003	132146.543
6000	7.845	79220.811	7.199	86251.937	5.075	137760.375
6400	7.939	83907.331	7.289	91057.344	5.141	143328.425
6800	8.020	88576.215	7.376	95840.700	5.203	148854.405
7200	8.101	93228.919	7.459	100603.824	5.268	154341.334
7600	8.179	97866.668	7.538	105348.294	5.329	159792.184
8000	8.254	102490.547	7.614	110075.493	5.387	165209.381

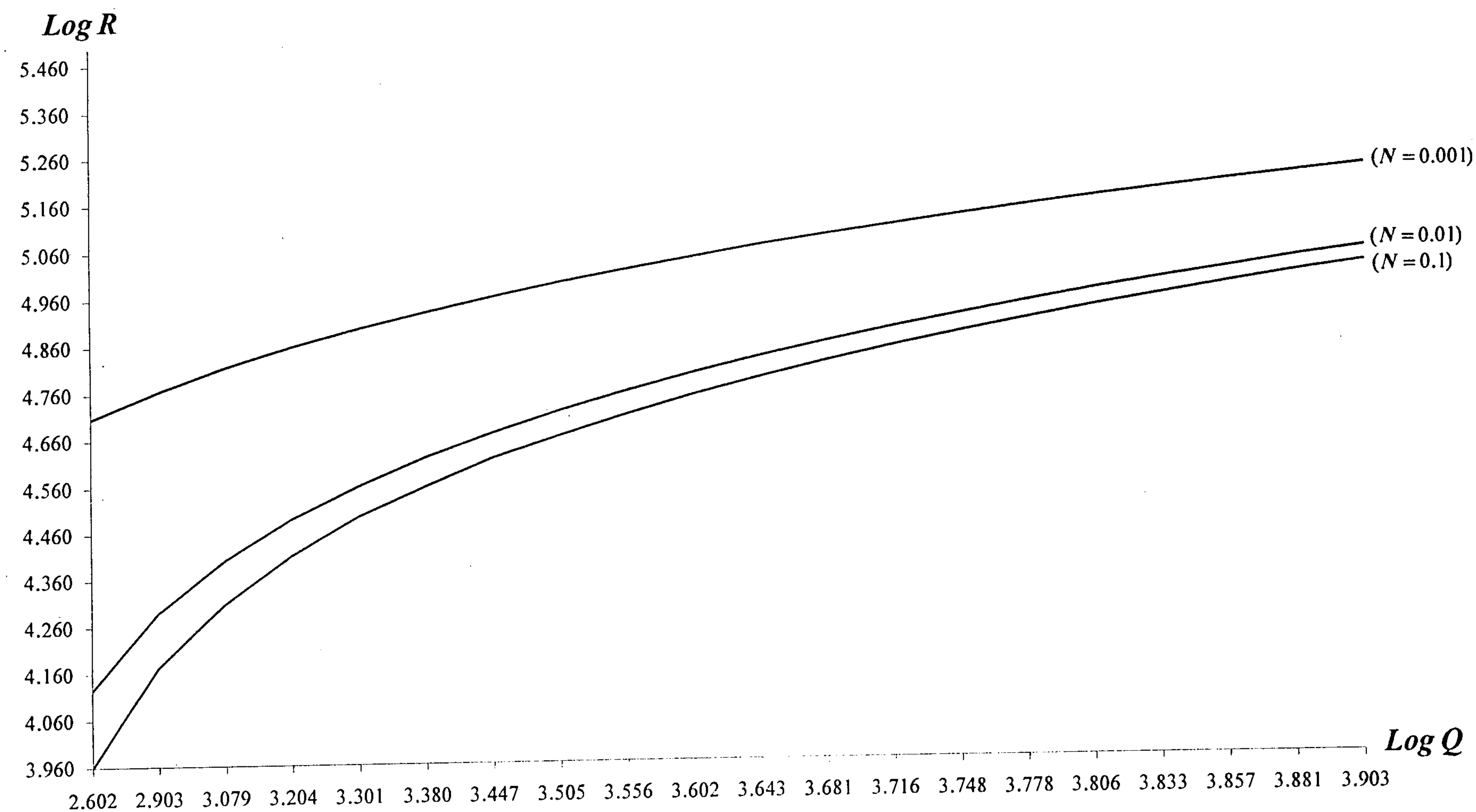

Figure 14. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 0$.

Table 14. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 1000$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
0	3.406	2521.382	3.269	6181.456	3.215	43167.927
400	4.884	9165.512	4.345	13364.580	3.473	50998.815
800	5.498	14939.023	4.888	19529.381	3.682	58321.930
1200	5.903	20384.297	5.267	25294.667	3.859	65296.956
1600	6.210	25645.891	5.563	30829.720	4.013	72014.152
2000	6.460	30785.211	5.807	36209.842	4.151	78530.424
2400	6.672	35835.230	6.016	41476.339	4.275	84884.383
2800	6.857	40816.128	6.199	46654.782	4.389	91103.647
3200	7.021	45741.341	6.363	51762.311	4.494	97208.784
3600	7.168	50620.363	6.512	56811.083	4.592	103215.597
4000	7.303	55460.205	6.648	61810.097	4.684	109136.542
4400	7.428	60266.225	6.773	66766.236	4.770	114981.643
4800	7.542	65042.630	6.890	71684.907	4.852	120759.110
5200	7.650	69792.799	6.999	76570.455	4.930	126475.772
5600	7.750	74519.498	7.101	81426.429	5.003	132137.369
6000	7.845	79225.022	7.198	86255.779	5.074	137748.804
6400	7.935	83911.307	7.289	91060.985	5.141	143314.281
6800	8.020	88580.002	7.376	95844.162	5.206	148837.453
7200	8.085	93232.777	7.458	100607.125	5.271	154321.544
7600	8.158	97870.359	7.578	105353.337	5.328	159769.272
8000	8.252	102493.850	7.607	110078.577	5.386	165183.206

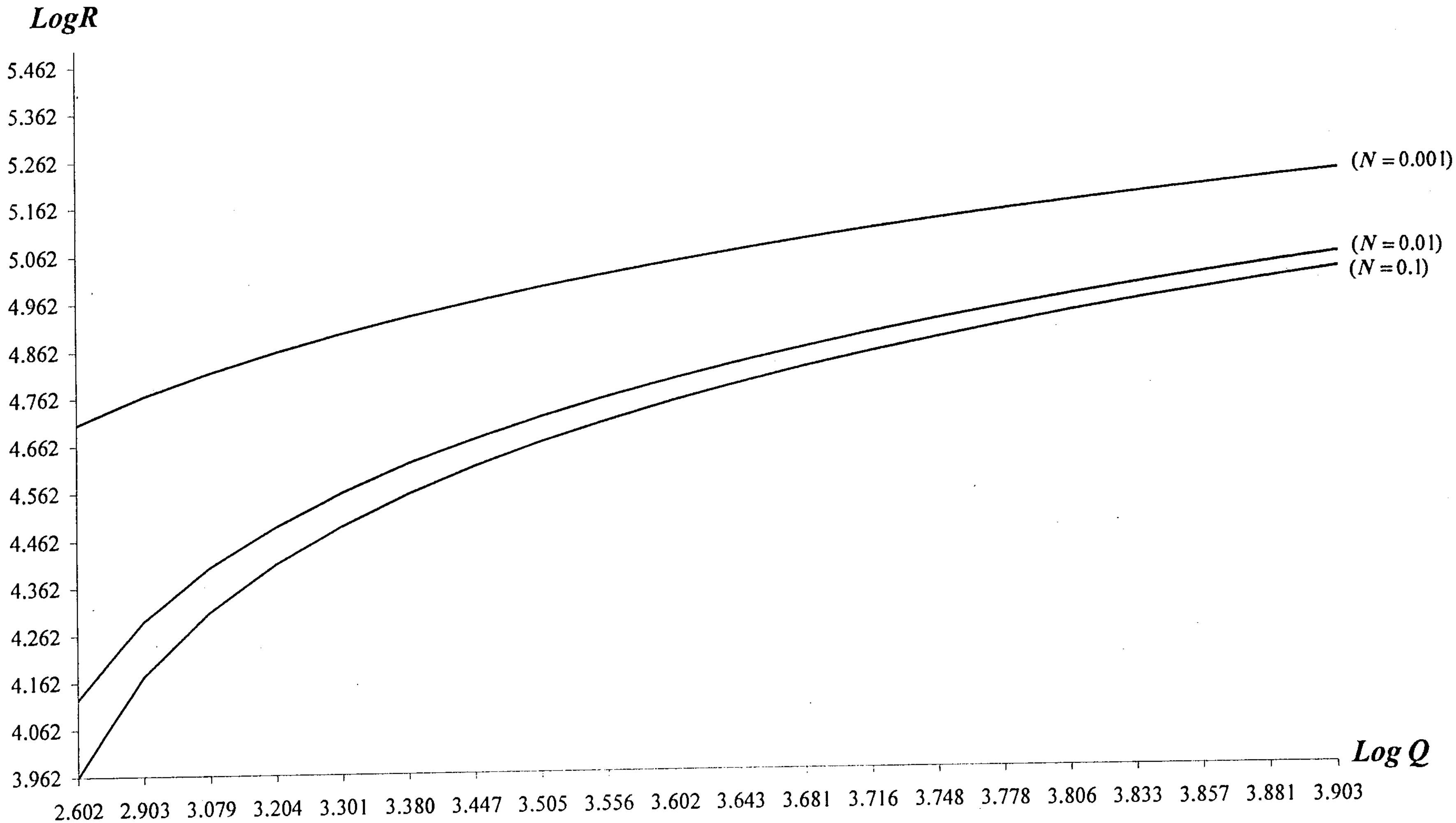

Figure 15. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T=1000$.

Table 15. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 10000$.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
0	4.506	4865.721	3.554	7361.873	3.221	43337.724
400	4.902	9563.665	4.365	13687.153	3.477	51119.365
800	5.495	15157.996	4.892	19718.236	3.684	58415.374
1200	5.898	20537.629	5.267	25429.788	3.860	65373.309
1600	6.205	25764.932	5.561	30935.745	4.014	72078.767
2000	6.456	30883.078	5.805	36297.560	4.151	78586.489
2400	6.668	35918.672	6.014	41551.443	4.276	84933.945
2800	6.853	40889.082	6.198	46720.647	4.389	91148.101
3200	7.017	45806.310	6.362	51821.104	4.495	97249.118
3600	7.165	50679.039	6.510	56864.281	4.592	103252.538
4000	7.300	55513.787	6.646	61858.752	4.684	109170.641
4400	7.425	60315.594	6.772	66811.124	4.770	115013.326
4800	7.540	65088.454	6.888	71726.620	4.852	120788.714
5200	7.647	69835.595	6.997	76609.451	4.930	126503.566
5600	7.748	74559.676	7.100	81463.073	5.003	132163.579
6000	7.843	79262.913	7.196	86290.365	5.074	137773.609
6400	7.933	83947.182	7.288	91093.756	5.141	143337.834
6800	8.018	88614.086	7.375	95875.317	5.206	148859.884
7200	8.100	93265.007	7.458	100636.833	5.268	154342.929
7600	8.177	97901.151	7.537	105379.855	5.328	159789.766
8000	8.251	102523.576	7.613	110105.740	5.386	165202.869

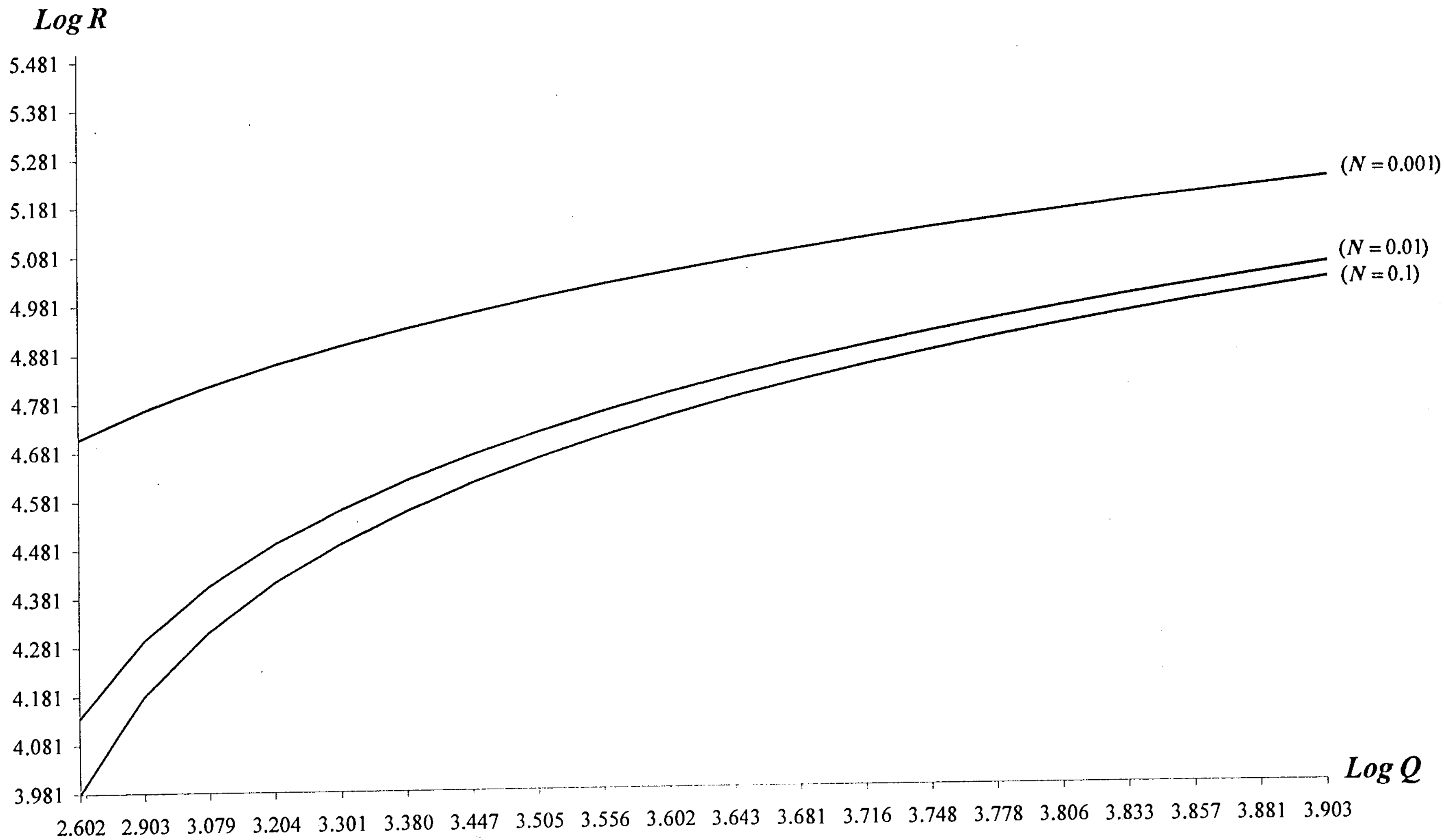

Figure 16. The relation between R and Q for the stationary convection case when both boundaries are rigid for T=10000.

Table 16. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 50000$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
0	6.008	11030.727	4.316	11555.702	3.248	44087.505
400	4.974	11298.227	4.446	15099.560	3.491	51653.600
800	5.484	16124.971	4.906	20554.014	3.693	58830.046
1200	5.876	21216.507	5.266	26029.000	3.866	65712.349
1600	6.183	26292.531	5.555	31406.250	4.018	72365.778
2000	6.435	31317.076	5.797	36686.954	4.154	78835.568
2400	6.649	36288.832	6.006	41884.906	4.278	85154.163
2800	6.836	41212.801	6.189	47013.124	4.391	91345.631
3200	7.002	46094.653	6.354	52082.201	4.496	97428.349
3600	7.151	50939.493	6.503	57100.546	4.593	103416.699
4000	7.288	55751.658	6.639	62074.851	4.685	109322.175
4400	7.413	60534.785	6.765	67010.504	4.771	115154.127
4800	7.529	65291.920	6.881	71911.900	4.852	120920.279
5200	7.637	70025.632	6.991	76782.670	4.930	126627.097
5600	7.739	74738.100	7.093	81625.850	5.004	132280.058
6000	7.834	79431.191	7.190	86444.008	5.074	137883.848
6400	7.925	84106.513	7.282	91239.336	5.141	143442.513
6800	8.007	88586.951	7.370	96013.725	5.206	148959.575
7200	8.092	93409.267	7.453	100768.816	5.268	154438.119
7600	8.170	98038.999	7.532	105506.046	5.328	159880.873
8000	8.245	102655.616	7.608	110226.682	5.386	165290.258

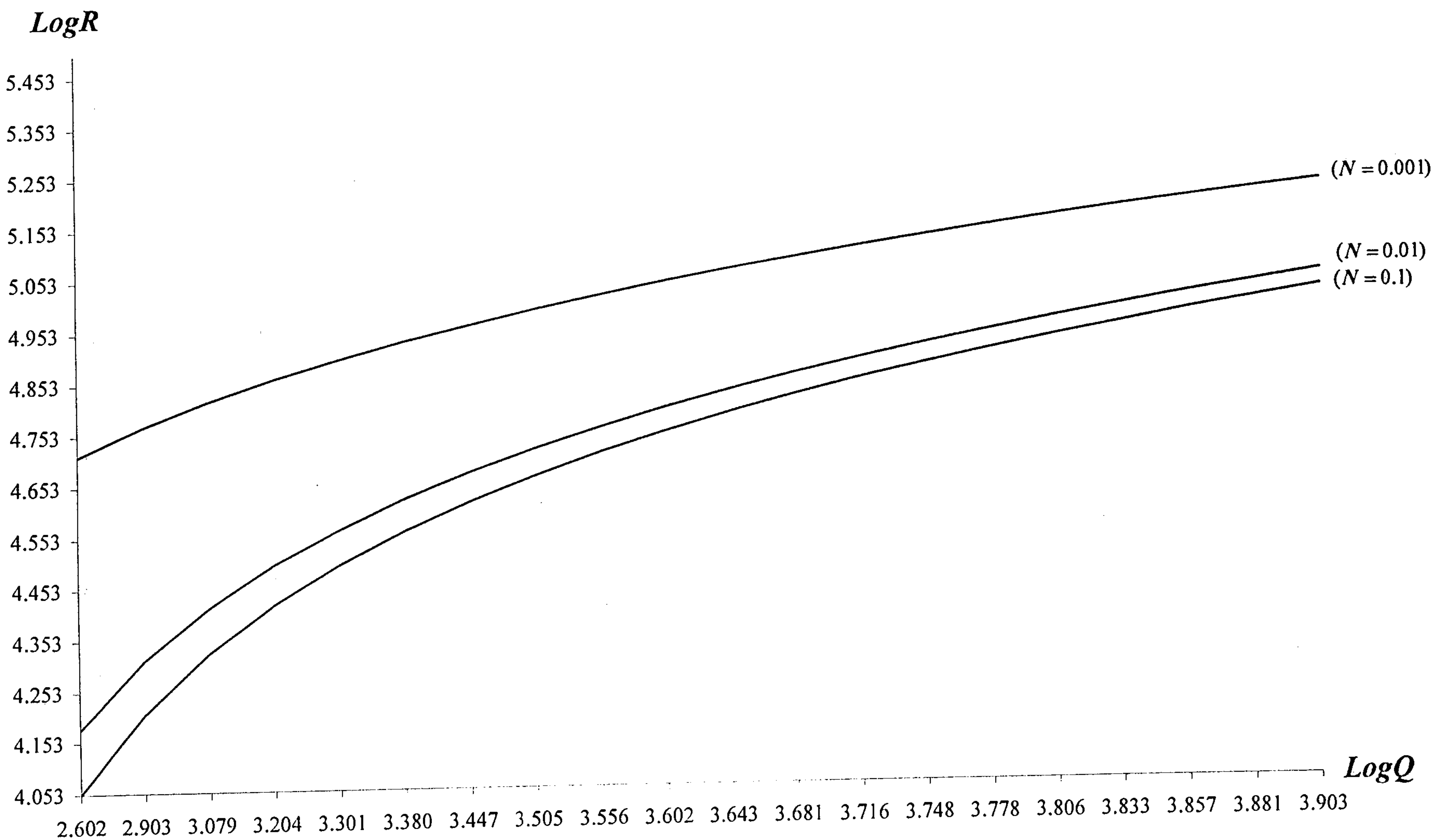

Figure 17. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 50000$.

Table 17. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 100000$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	6.835	16508.903	4.879	15773.005	3.281	45014.023
400	5.056	13399.184	4.538	16822.443	3.510	52317.935
800	5.473	17320.516	4.923	21590.961	3.704	59346.952
1200	5.852	22059.468	5.265	26775.076	3.874	66135.450
1600	6.157	26948.783	5.548	31992.785	4.024	72724.163
2000	6.411	31857.419	5.788	37172.652	4.158	79146.692
2400	6.627	36749.983	5.996	42300.976	4.281	85429.293
2800	6.816	41616.275	6.179	47378.134	4.393	91592.450
3200	6.983	46454.158	6.344	52408.103	4.497	97652.323
3600	7.134	51264.313	6.493	57395.489	4.595	103621.854
4000	7.272	56048.379	6.630	62344.647	4.686	109511.558
4400	7.398	60808.253	6.756	67259.448	4.772	115330.101
4800	7.516	65545.809	6.873	72143.256	4.853	121084.713
5200	7.625	70262.794	6.983	76998.979	4.930	126781.493
5600	7.727	74960.796	7.083	81647.642	5.004	132425.643
6000	7.823	79641.244	7.180	86452.697	5.074	138021.634
6400	7.914	84305.415	7.276	91421.159	5.142	143573.348
6800	8.001	88954.452	7.363	96186.596	5.206	149084.174
7200	8.083	93589.381	7.446	100933.668	5.268	154557.095
7600	8.161	98211.119	7.526	105663.670	5.328	159994.749
8000	8.236	102820.494	7.602	110377.755	5.386	165399.486

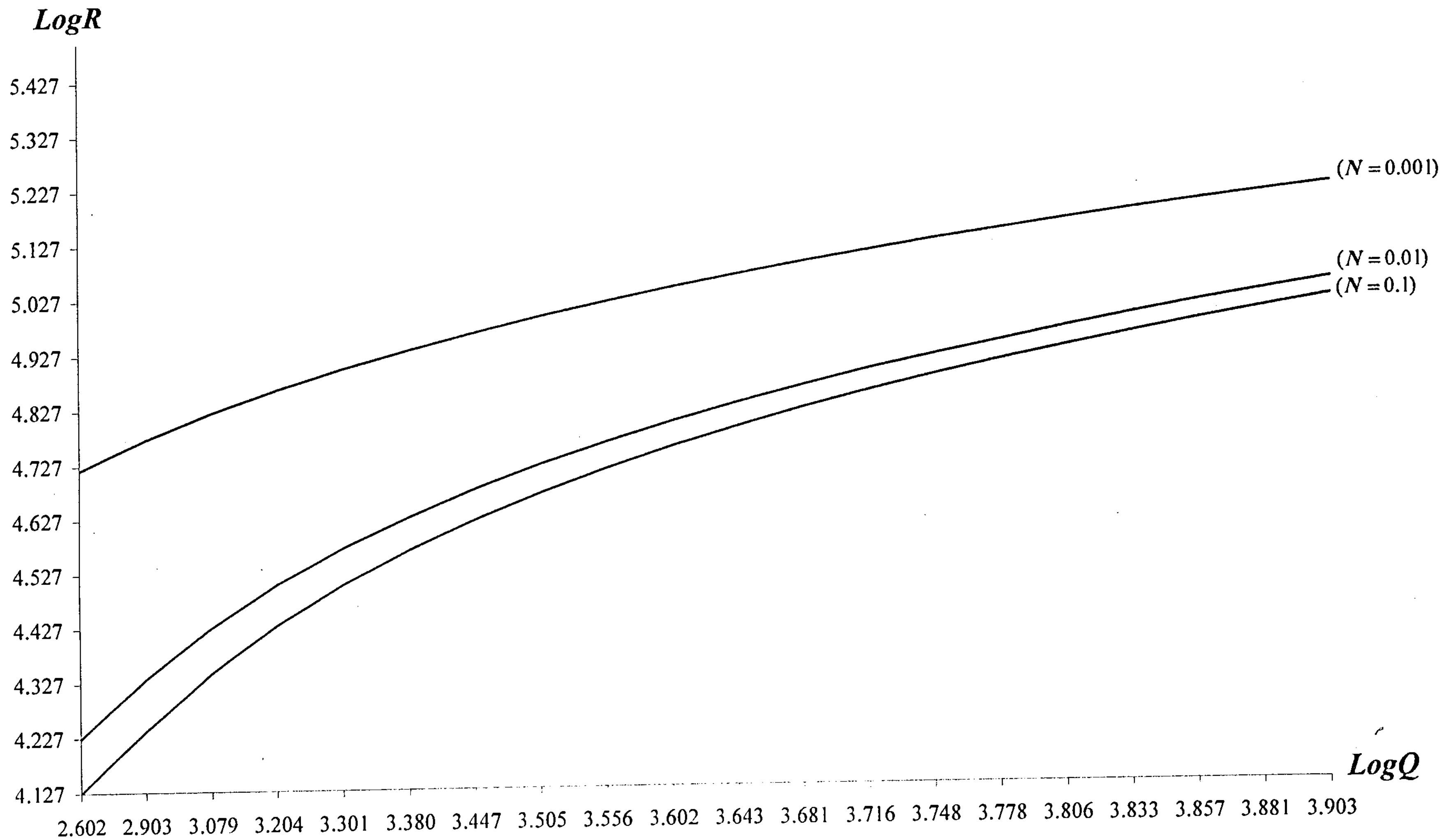

Figure 18. The relation between R and Q for the stationary convection case when both boundaries are rigid for $T = 100000$.

Table 18. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 0$.

	(N=0.1)		(N=0.01)		(N=0.01)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
0	-	-	-	-	-	-
400	4.565	8364.743	-	-	-	-
800	5.110	11251.520	4.769	18767.749	-	-
1200	5.479	13757.375	5.112	21873.353	-	-
1600	5.762	16048.260	5.387	24706.797	-	-
2000	5.993	18195.301	5.618	27352.885	-	-
2400	6.190	20236.986	5.817	29859.352	-	-
2800	6.363	22196.995	5.994	32256.334	-	-
3200	6.516	24091.161	6.153	34564.341	-	-
3600	6.654	25930.704	6.297	36798.101	-	-
4000	6.780	27723.928	6.430	38968.623	-	-
4400	6.896	29477.187	6.552	41084.398	-	-
4800	7.004	31195.477	6.667	43152.145	-	-
5200	7.105	32882.812	6.774	45177.289	5.021	124862.907
5600	7.167	34543.910	6.875	47164.289	5.090	128127.520
6000	7.288	36177.200	6.970	49116.863	5.140	131331.739
6400	7.373	37789.279	7.060	51038.150	5.219	134476.299
6800	7.453	39380.671	7.146	52930.831	5.280	137570.102
7200	7.529	40953.060	7.229	54797.217	5.339	140615.567
7600	7.602	42507.906	7.307	56639.319	5.397	143616.118
8000	7.670	44046.490	7.382	58458.897	5.452	146574.782

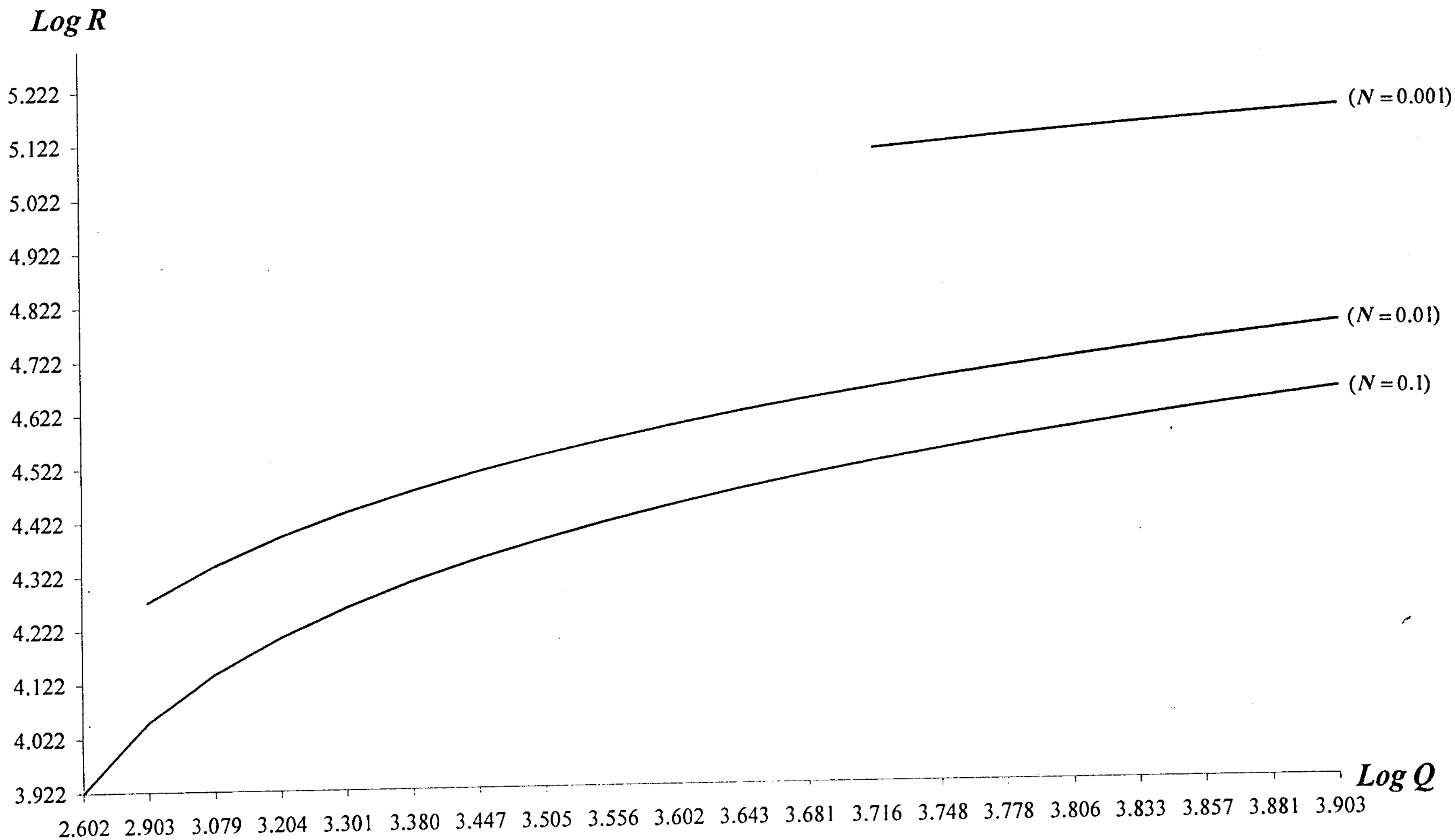

Figure 19. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 0$.

Table 19. The relation between R and Q for the overstability case when both boundaries are rigid for T =1000.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	-	-	-	-	-	-
400	4.576	8494.995	-	-	-	-
800	5.111	11329.091	4.771	18823.294	-	-
1200	5.478	13812.570	5.112	21914.194	-	-
1600	5.760	16090.947	5.386	24738.852	-	-
2000	5.992	18229.964	5.617	27379.083	-	-
2400	6.189	20266.051	5.817	29881.365	-	-
2800	6.361	22221.927	5.993	32275.207	-	-
3200	6.514	24112.915	6.152	34580.771	-	-
3600	6.652	25949.936	6.296	36812.577	-	-
4000	6.778	27741.110	6.429	38981.500	-	-
4400	6.895	29492.670	6.551	41095.944	-	-
4800	7.003	31209.530	6.666	43162.566	-	-
5200	7.103	32895.643	6.773	45186.747	5.021	124870.559
5600	7.198	34554.253	6.874	47172.914	5.090	128134.750
6000	7.287	36188.056	6.969	49124.760	5.156	131337.106
6400	7.371	37799.325	7.059	51045.406	5.219	134482.793
6800	7.452	39390.000	7.146	52937.519	5.280	137576.274
7200	7.528	40961.749	7.228	54803.399	5.339	140621.443
7600	7.601	42516.020	7.306	56645.045	5.397	143621.720
8000	7.671	44054.083	7.382	58464.212	5.452	146580.130

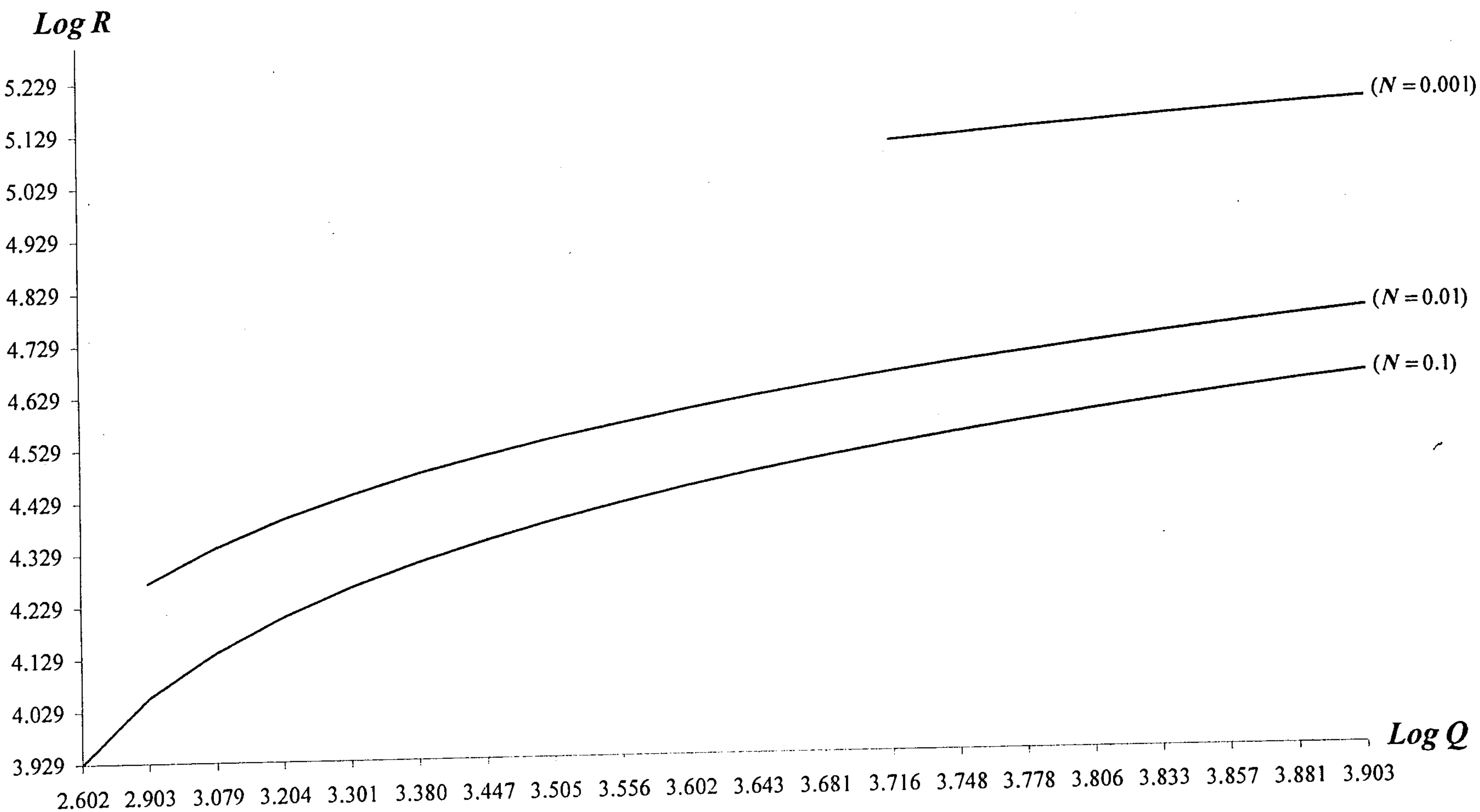

Figure 20. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 1000$.

Table 20. The relation between R and Q for the overstability case when both boundaries are rigid for T =10000.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	-	-	-	-	-	-
400	-	-	-	-	-	-
800	5.124	12008.863	4.787	19320.816	-	-
1200	5.472	14300.220	5.115	22281.230	-	-
1600	5.748	16469.842	5.383	25027.380	-	-
2000	5.978	18538.576	5.611	27615.118	-	-
2400	6.175	20525.385	5.809	30079.812	-	-
2800	6.347	22444.748	5.985	32445.413	-	-
3200	6.500	24307.574	6.143	34728.987	-	-
3600	6.639	26122.204	6.286	36943.194	-	-
4000	6.766	27895.146	6.420	39097.713	-	-
4400	6.883	29631.571	6.543	41200.161	-	-
4800	6.991	31335.665	6.657	43256.641	-	-
5200	7.092	33010.869	6.765	45272.139	5.021	124939.433
5600	7.167	34660.619	6.866	47250.792	5.090	128199.819
6000	7.277	36285.618	6.961	49196.074	5.156	131398.717
6400	7.362	37889.641	7.052	51110.940	5.219	134541.244
6800	7.442	39473.888	7.138	52997.927	5.280	137631.827
7200	7.519	41039.898	7.221	54859.230	5.339	140674.327
7600	7.592	42589.016	7.299	56696.768	5.396	143672.139
8000	7.662	44122.426	7.375	58512.229	5.452	146628.265

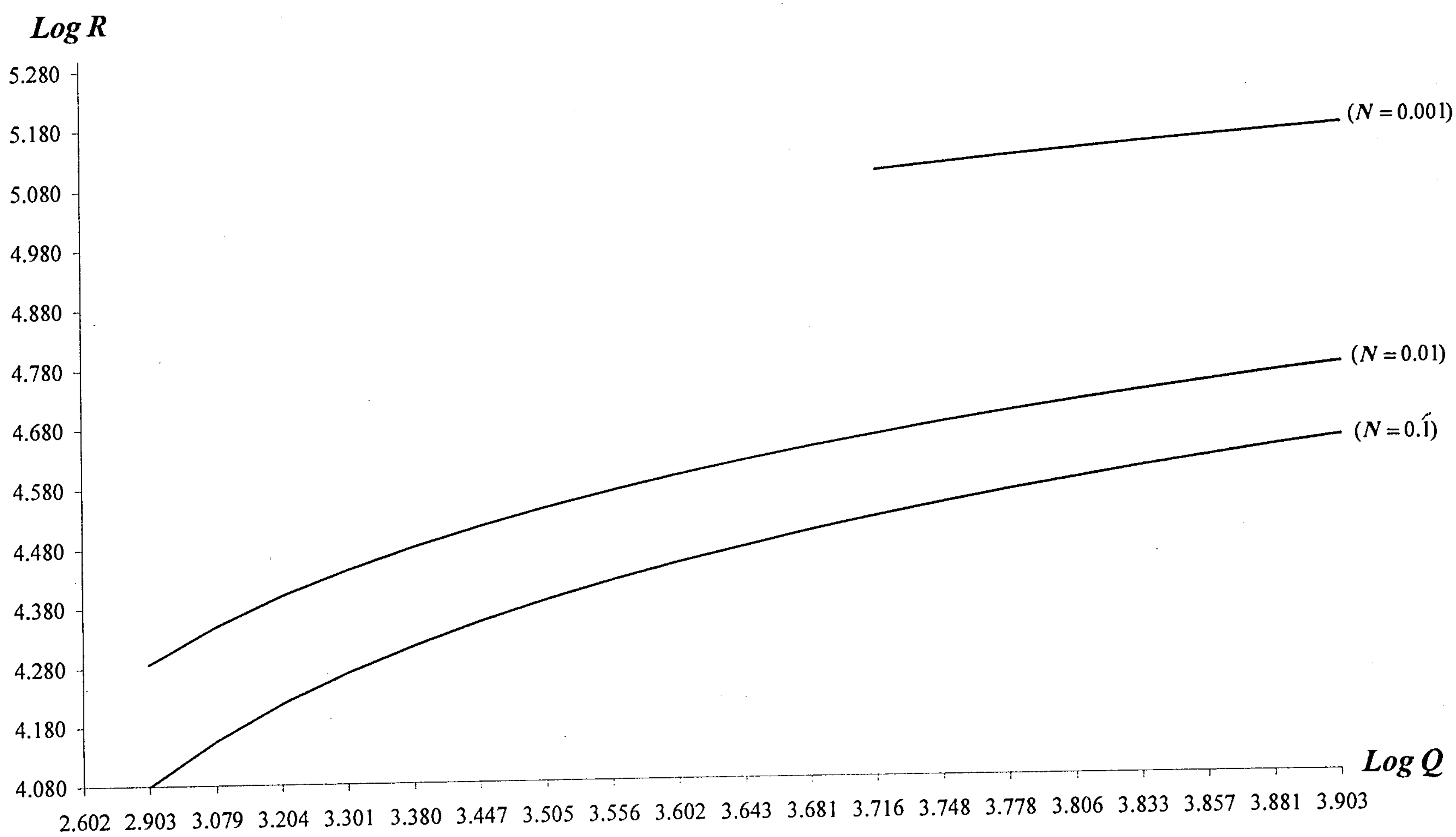

Figure 21. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 10000$.

Table 21. The relation between R and Q for the overstability case when both boundaries are rigid for T =50000.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	-	-	-	-	-	-
400	-	-	-	-	-	-
800	5.205	14823.705	-	-	-	-
1200	5.467	16360.500	5.133	23901.057	-	-
1600	5.712	18088.200	5.375	26310.099	-	-
2000	5.929	19866.571	5.589	28668.988	-	-
2400	6.121	21647.618	5.780	30968.345	-	-
2800	6.292	23413.287	5.952	33208.998	-	-
3200	6.446	25156.800	6.109	35394.888	-	-
3600	6.586	26876.047	6.252	37530.673	-	-
4000	6.715	28570.953	6.385	39620.890	-	-
4400	6.833	30242.333	6.507	41669.684	-	-
4800	6.943	31891.370	6.622	43680.741	-	-
5200	7.046	33519.366	6.730	45657.308	5.022	125245.600
5600	7.142	35127.623	6.832	47602.241	5.089	128445.469
6000	7.234	36717.382	6.928	49518.049	5.156	131672.647
6400	7.320	38289.806	7.020	51406.943	5.219	134801.148
6800	7.402	39845.971	7.107	53270.882	5.280	137878.864
7200	7.480	41386.863	7.190	55111.605	5.339	140909.516
7600	7.554	42913.386	7.269	56930.660	5.396	143896.381
8000	7.625	44426.365	7.345	58729.438	5.451	146842.360

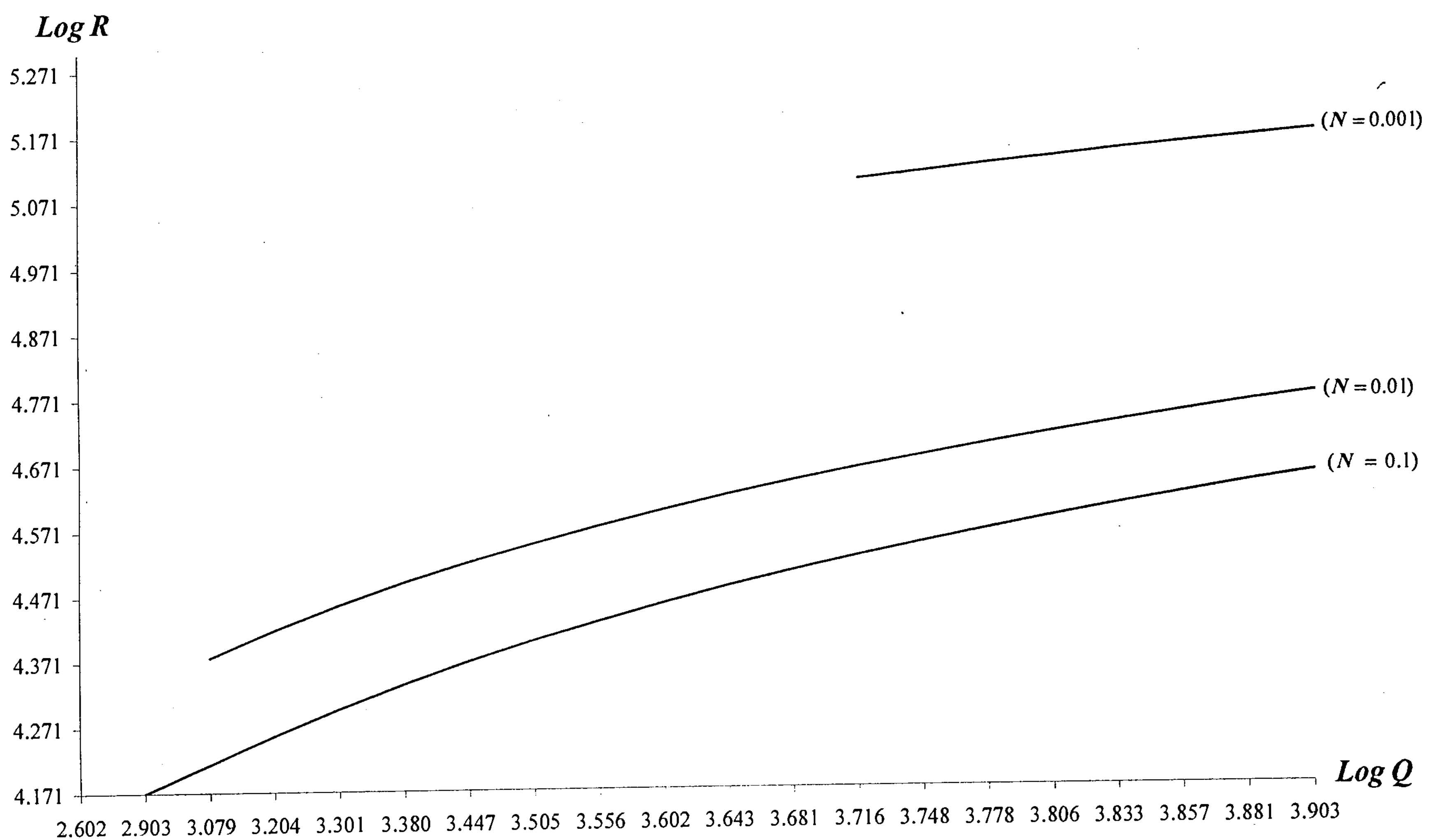

Figure 22. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 50000$.

Table 22. The relation between R and Q for the overstability case when both boundaries are rigid for T =100000.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
0	-	-	-	-	-	-
400	-	-	-	-	-	-
800	-	-	-	-	-	-
1200	5.495	18821.733	5.169	25898.624	-	-
1600	5.698	20044.513	5.379	27911.412	-	-
2000	5.894	21483.098	5.575	29994.485	-	-
2400	6.075	23020.327	5.756	32091.505	-	-
2800	6.241	24602.485	5.922	34177.687	-	-
3200	6.393	26202.781	6.074	36241.941	-	-
3600	6.532	27807.063	6.216	38279.560	-	-
4000	6.660	29407.588	6.347	40288.954	-	-
4400	6.780	31000.060	6.469	42270.094	-	-
4800	6.891	32582.124	6.584	44223.729	-	-
5200	6.995	34152.552	6.692	46150.982	5.024	125628.431
5600	7.093	35710.787	6.794	48053.127	5.092	128850.895
6000	7.185	37256.677	6.891	49931.480	5.157	132015.297
6400	7.273	38790.316	6.983	51787.333	5.220	135126.303
6800	7.356	40311.945	7.070	53621.921	5.281	138187.965
7200	7.435	41821.891	7.154	55436.409	5.339	141203.832
7600	7.511	43320.529	7.234	57231.889	5.396	144177.031
8000	7.583	44808.256	7.337	59010.676	5.451	147110.341

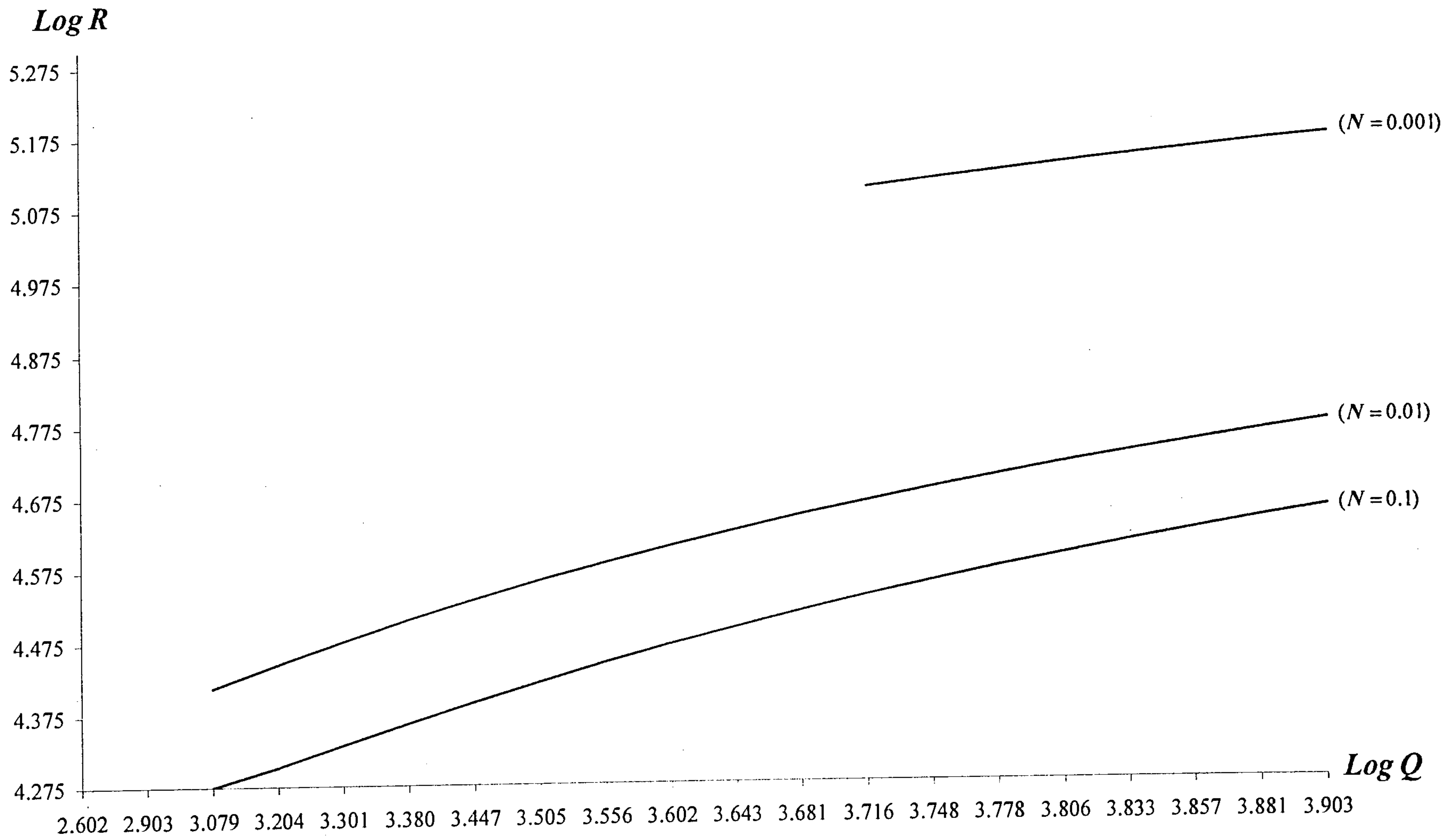

Figure 23. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 100000$.

Table 23. A comparison between the cases of overstability and stationary convection when both boundaries are rigid for $T = 10000$.

	(*N*=0.01)				(*N*=0.001)			
	Overstability		Stationary		Overstability		Stationary	
Q	a	R	a	R	a	R	a	R
0	-	-	3.554	7361.873	-	-	3.221	43337.724
400	-	-	4.365	13687.153	-	-	3.477	51119.365
800	4.787	19320.816	4.892	19718.236	-	-	3.684	58415.374
1200	5.115	22281.230	5.267	25429.788	-	-	3.860	65373.309
1600	5.383	25027.380	5.561	30935.745	-	-	4.014	72078.767
2000	5.611	27615.118	5.805	36297.560	-	-	4.151	78586.489
2400	5.809	30079.812	6.014	41551.443	-	-	4.276	84933.945
2800	5.985	32445.413	6.198	46720.647	-	-	4.389	91148.101
3200	6.143	34728.987	6.362	51821.104	-	-	4.495	97249.118
3600	6.286	36943.194	6.510	56864.281	-	-	4.592	103252.538
4000	6.420	39097.713	6.646	61858.752	-	-	4.684	109170.641
4400	6.543	41200.161	6.772	66811.124	-	-	4.770	115013.326
4800	6.657	43256.641	6.888	71726.620	-	-	4.852	120788.714
5200	6.765	45272.139	6.997	76609.451	5.021	124939.433	4.930	126503.566
5600	6.866	47250.792	7.100	81463.073	5.090	128199.819	5.003	132163.579
6000	6.961	49196.074	7.196	86290.365	5.156	131398.717	5.074	137773.609
6400	7.052	51110.940	7.288	91093.756	5.219	134541.244	5.141	143337.834
6800	7.138	52997.927	7.375	95875.317	5.280	137631.827	5.206	148859.884
7200	7.221	54859.230	7.458	100636.833	5.339	140674.327	5.268	154342.929
7600	7.299	56696.768	7.537	105379.855	5.396	143672.139	5.328	159789.766
8000	7.375	58512.229	7.613	110105.740	5.452	146628.265	5.386	165202.869

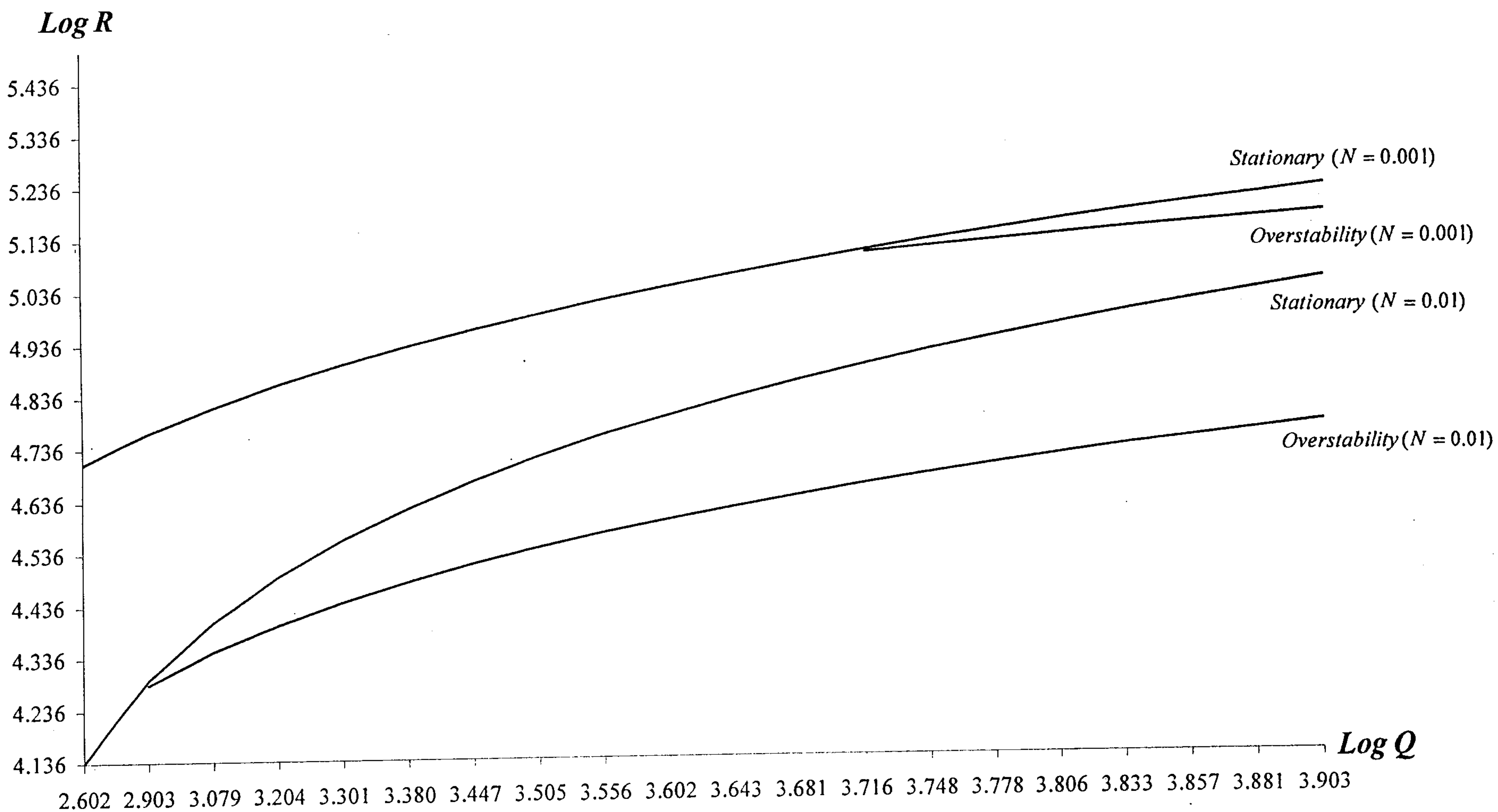

Figure 24. A comparison between the cases of overstability and stationary convection when both boundaries are rigid for $T = 10000$.

Chapter Four

Benard convection in a horizontal porous layer permeated by a non-linear magnetic fluid under the influence of both magnetic field and Coriolis forces

4.1 Mathematical formulation

In the previous chapter we have discussed Benard convection in a horizontal porous layer permeated by a conducting fluid under the influence of both magnetic field and Coriolis forces when the relation between the magnetic field $\underline{H}$ and the magnetic induction $\underline{B}$ is linear. Here we shall use a non-linear constitutive relationship of the form (see Roberts (1981)).

$$H_i = \rho \frac{\partial \psi^*}{\partial B_i}, \qquad (4.1.1)$$

where $\psi^* = \psi^*(\rho, \underline{B})$ is the internal energy function. Since ψ^* must be invariant then the dependence of ψ^* on $\underline{B}$ is reduced to $\psi^* = \psi(\rho, B)$. Thus from (4.1.1)

$$H_i = \rho \phi B_i$$

where

$$\phi = \frac{1}{B} \frac{\partial \psi}{\partial B} \qquad (4.1.2)$$

is the susceptibility. According to this non-linear relationship the term $(\underline{J} \times \underline{B})$ in the equation of motion (3.1.4) can be rewritten as

$$
\begin{aligned}
(\underline{J}\times\underline{B})_i &= (curl\,\underline{H}\times\underline{B})_i \\
&= e_{ijk}(curl\,\underline{H})_j B_k \\
&= e_{ijk}e_{jrs}H_{s,r}B_k \\
&= (\delta_{kr}\delta_{is}-\delta_{ks}\delta_{ir})H_{s,r}B_k \\
&= H_{i,k}B_k - H_{k,i}B_k \\
&= H_{i,k}B_k - (H_k B_k)_{,i} + H_k B_{k,i} \\
&= (\rho\phi B_i)_{,k}B_k - (\rho\phi B^2)_{,i} + \rho\phi B B_{,i}
\end{aligned}
$$

Using (4.1.2) we can show that

$$
(\underline{J}\times\underline{B})_i = (\rho\phi B_i)_{,k}B_k - (\rho B\psi_B - \rho\psi)_{,i}
$$

but

$$
\begin{aligned}
B\psi_B - \psi &= B^2\phi - \int\phi B\,dB \\
&= B^2\phi - \frac{1}{2}\left[\phi B^2 - \int B^2\phi_B\,dB\right] \\
&= \frac{1}{2}\left(B^2\phi + \int B^2\phi\,dB\right)
\end{aligned}
$$

$$
\therefore (\underline{J}\times\underline{B})_i = (\rho\phi B_i)_{,k}B_k - \frac{\rho}{2}\left(B^2\phi + \int B^2\phi_B\,dB\right)_{,i}.
$$

Thus the equation of motion (3.1.4) is modified to

$$
\rho\dot{\upsilon}_i = -P_{,i} + \rho\,\nu\nabla^2\upsilon_i + \rho\,g_i + \left(\rho\phi B_i\right)_{,k}B_k - \frac{\mu'}{k_1}\upsilon_i + 2\rho(\underline{\upsilon}\times\underline{\Omega})_i,
$$

where

$$
P = p + \frac{\rho}{2}\left(B^2\phi + \int B^2\phi_B\,dB\right) - \frac{1}{2}\rho_0\left|\underline{\Omega}\times\underline{r}\right|^2.
$$

If we now make the Boussinesq approximation then the governing field equations become

$$
\begin{aligned}
&\upsilon_{i,i} = 0,\\
&\frac{D\upsilon_i}{Dt} = -\left(\frac{P}{\rho_o}\right)_{,i} + \nu\nabla^2 \upsilon_i - g(1-\alpha\theta)\delta_{i3} + (\phi B_i)_{,k} B_k - \frac{\nu}{k_1}\upsilon_i + 2e_{ijk}\upsilon_j \Omega_k,\\
&\frac{D\theta}{Dt} = k\nabla^2 \theta,\\
&\frac{\partial B_i}{\partial t} = B_j \upsilon_{i,j} - \upsilon_j B_{i,j} - \eta e_{ijk} J_{k,j},
\end{aligned}
\tag{4.1.3}
$$

together with the Maxwell equations

$$
\begin{aligned}
&div\,\underline{B} = B_{i,i} = 0,\\
&curl\,\underline{H} = e_{ijk} H_{k,j} = J_i,\\
&curl\,\underline{E} = e_{ijk} E_{k,j} = -\frac{\partial B_i}{\partial t}.
\end{aligned}
\tag{4.1.4}
$$

Equation (4.1.3) and (4.1.4) have a steady solution in which

$$
\begin{aligned}
&\underline{\upsilon} = 0,\\
&\theta = \theta(x_3) = T_o - \beta x_3,\\
&P = P(x_3),\\
&\underline{B} = (0,0,B), \qquad B = \text{constant},\\
&\underline{J} = 0,\\
&\phi = \phi(B).
\end{aligned}
\tag{4.1.5}
$$

Suppose that the initial state described by equations (4.1.5) is slightly perturbed so that

$$
\begin{aligned}
&\underline{\upsilon} = \underline{0} + \varepsilon^* \underline{\upsilon}^*, \qquad \theta = T_o - \beta x_3 + \varepsilon^* \theta^* \qquad P = P + \varepsilon^* P^*,\\
&\underline{B} = (0,0,B) + \varepsilon^* \underline{b}^*, \qquad \underline{J} = \underline{0} + \varepsilon^* \underline{J}^*, \qquad \phi = \phi + \varepsilon^* b_3^* \phi_B ,
\end{aligned}
$$

where ε^* is the perturbation parameter and $\underline{\upsilon}^*, \theta^*, P^*, \underline{b}^*$ and $\underline{J}^*$ are respectively the linear perturbation of velocity, temperature, pressure, magnetic induction and current density about their values described in (4.1.5).

The linear perturbation of ϕ about its value $(4.1.5)_6$ can be obtained in the following way :

$$
\begin{aligned}
B &= (B_i B_i)^{1/2} \\
&= \left\{\left[(0,0,B)+\varepsilon^* \underline{b}^*\right].\left[(0,0,B)+\varepsilon^* \underline{b}^*\right]\right\}^{1/2} \\
&\approx \left[B^2 + 2\varepsilon^* (0,0,B).\underline{b}^*\right]^{1/2} \\
&= B\left[1+\frac{2\varepsilon^* (0,0,B).\underline{b}^*}{B^2}\right]^{1/2} \\
&\approx B\left[1+\frac{\varepsilon^* (0,0,B).\underline{b}^*}{B^2}+\ldots\right] \\
&\approx B\left[1+\frac{\varepsilon^* (0,0,B).\underline{b}^*}{B^2}\right] \\
&= B+\frac{\varepsilon^* (0,0,B).\underline{b}^*}{B} \; .
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\phi &= \phi\left(B+\frac{\varepsilon^* (0,0,B).\underline{b}^*}{B}\right) \\
&\approx \phi(B)+\frac{\varepsilon^* (0,0,B).\underline{b}^*}{B}\phi_B(B)+\ldots \\
&\approx \phi(B)+\varepsilon^* b_3^* \phi_B(B) \; ; \qquad \underline{b}^* = (b_1^*, b_2^*, b_3^*).
\end{aligned}
$$

We may verify that the linearized versions of equations (4.1.3) and (4.1.4) are

$$
\begin{gathered}
\frac{\partial \upsilon_i^*}{\partial t} = -\left(\frac{P^*}{\rho_o}\right)_{,i} + \nu\nabla^2\upsilon_i^* + g\alpha\theta^*\delta_{i3} + B\phi b_{i,3}^* + B^2\phi_B b_{3,3}^*\delta_{i3} - \frac{\nu}{k_1}\upsilon_i^* + 2e_{ijk}\upsilon_j^*\Omega_k, \\
\upsilon_{i,i}^* = 0, \\
\frac{\partial \theta^*}{\partial t} - \beta\upsilon_3^* = k\nabla^2\theta^*, \qquad (4.1.6) \\
b_{i,i}^* = 0, \\
\frac{\partial b_i^*}{\partial t} = B\upsilon_{i,3}^* - \eta e_{ijk} J_{k,j}^*, \\
J_i^* = e_{ijk}\left(\rho\phi b_k^* + \rho B\phi_B b_3^* \delta_{k3}\right)_{,j} .
\end{gathered}
$$

Now we introduce dimensionless variables $\hat{x}_i, \hat{\upsilon}_i, \hat{t}, \hat{J}_i, \hat{\theta}, \hat{P}$ and $\hat{b}_i$ such that

$$x_i^* = d\hat{x}_i, \quad \upsilon_i^* = \frac{k}{d}\hat{\upsilon}_i, \quad t = \frac{d^2}{\nu}\hat{t}, \quad J_i^* = \frac{k\nu\rho_o}{Bd^3}\hat{J}_i,$$

$$\theta^* = \frac{k}{d}\sqrt{\frac{\nu|\beta|}{k\alpha g}}\hat{\theta}, \quad P^* = \frac{k\nu\rho_o}{d^2}\hat{P} \quad \text{and} \quad b_i^* = \frac{\kappa\nu}{B\phi d^2}\hat{b}_i.$$

After this non-dimensionalization, equations (4.1.6) simplify to

$$\begin{aligned}
&\upsilon_{i,i} = 0,\\
&\frac{\partial \upsilon_i}{\partial t} = -P_{,i} + \nabla^2 \upsilon_i + \sqrt{R}\theta\delta_{i3} + b_{i,3} + \varepsilon b_{3,3}\delta_{i3} - \frac{1}{N}\upsilon_i + \sqrt{T}e_{ijk}\upsilon_j\delta_{k3},\\
&P_r\frac{\partial \theta}{\partial t} + H\sqrt{R}\upsilon_3 = \nabla^2 \theta,\\
&b_{i,i} = 0,\\
&P_m\frac{\partial b_i}{\partial t} = Q\upsilon_{i,3} - e_{ijk}J_{k,j},\\
&J_i = e_{ijk}b_{k,j} + e_{ijk}\varepsilon b_{3,j}\delta_{k3},
\end{aligned} \tag{4.1.7}$$

where the (^) superscript has been dropped but all the variables are now non-dimensional and where the non-dimensional numbers $R, N, P_r, P_m, \varepsilon, Q$ and T are given by

$$R = \frac{\alpha g|\beta|d^4}{\nu k}, \qquad N = \frac{k_1}{d^2},$$

$$P_r = \frac{\nu}{k}, \qquad P_m = \frac{\nu\mu}{\eta}, \qquad \varepsilon = \frac{B}{\phi}\phi_B,$$

$$Q = \frac{B^2 d^2}{\rho_o \nu\eta}, \qquad T = \frac{4\Omega^2 d^4}{\nu^2}.$$

Equations $(4.1.7)_5$ and $(4.1.7)_6$ can be combined to obtain

$$\begin{aligned}
P_m \frac{\partial b_i}{\partial t} &= Q\upsilon_{i,3} - e_{ijk} J_{k,j} \\
&= Q\upsilon_{i,3} - e_{ijk} e_{krs}(b_{s,r} + \varepsilon b_{3,r}\delta_{s3})_{,j} \\
&= Q\upsilon_{i,3} - (\delta_{ir}\delta_{js} - \delta_{is}\delta_{jr})(b_{s,rj} + \varepsilon b_{3,rj}\delta_{s3}) \\
&= Q\upsilon_{i,3} - (b_{j,ij} - b_{i,jj} + \varepsilon b_{3,ij}\delta_{j3} - \varepsilon b_{3,jj}\delta_{i3}) \\
&= Q\upsilon_{i,3} + \nabla^2 b_i + \varepsilon \nabla^2 b_3 \delta_{i3} - \varepsilon b_{3,i3}.
\end{aligned} \tag{4.1.8}$$

4.2 The eigenvalue problem

Following the same procedures as in the previous chapter, we shall define the variables w, b, J, ξ and z as

$$w = \upsilon_3, \quad b = b_3, \quad J = J_3, \quad \xi = \xi_3, \quad z = x_3.$$

When we take the curl of equations $(4.1.7)_2$ and (4.1.8) we obtain

$$\begin{aligned}
\frac{\partial \xi_i}{\partial t} &= \nabla^2 \xi_i + \sqrt{R}\, e_{ijk}\theta_{,j}\delta_{k3} + e_{ijk} b_{k,3j} + \varepsilon\, e_{ijk} b_{3,3j}\delta_{k3} - \frac{1}{N}\xi_i + \sqrt{T}\,\upsilon_{i,3}, \\
P_m \frac{\partial J_i}{\partial t} &- \varepsilon P_m e_{ijk} \frac{\partial b_{3,j}}{\partial t}\delta_{k3} = Q\xi_{i,3} + J_{i,jj}.
\end{aligned} \tag{4.2.1}$$

Taking the curl of equation $(4.2.1)_1$ once again, we obtain

$$\begin{aligned}
\frac{\partial \nabla^2 \upsilon_i}{\partial t} &= \nabla^4 \upsilon_i - \sqrt{R}\left(\frac{\partial^2 \theta}{\partial x_3\, \partial x_i} - \frac{\partial^2 \theta}{\partial x_j^2}\delta_{i3}\right) + \nabla^2 b_{i,3} + \varepsilon \nabla^2 b_{3,3}\delta_{i3} - \varepsilon\, b_{3,33i} \\
&\quad - \frac{1}{N}\nabla^2 \upsilon_i - \sqrt{T}\,\xi_{i,3}
\end{aligned} \tag{4.2.2}$$

The third components of equations (4.2.1), (4.2.2) and (4.1.8) yield

$$\frac{\partial \xi}{\partial t} = \nabla^2 \xi + \frac{\partial J}{\partial z} - \frac{1}{N}\xi + \sqrt{T}\frac{\partial w}{\partial z},$$

$$P_m \frac{\partial J}{\partial t} = Q\frac{\partial \xi}{\partial z} + \nabla^2 J, \tag{4.2.3}$$

$$\frac{\partial \nabla^2 w}{\partial t} = \nabla^4 w + \sqrt{R}(\frac{\partial^2 \theta}{\partial x_1^2} + \frac{\partial^2 \theta}{\partial x_2^2}) + \nabla^2\left(\frac{\partial b}{\partial z}\right) + \varepsilon\frac{\partial}{\partial z}\left(\frac{\partial^2 b}{\partial x_1^2} + \frac{\partial^2 b}{\partial x_2^2}\right) - \frac{1}{N}\nabla^2 w - \sqrt{T}\frac{\partial \xi}{\partial z}$$

$$P_m \frac{\partial b}{\partial t} = Q\frac{\partial w}{\partial z} + \nabla^2 b + \varepsilon\left(\frac{\partial^2 b}{\partial x_1^2} + \frac{\partial^2 b}{\partial x_2^2}\right).$$

Now we look for a solution of the form

$$\Phi = \Phi(z)\exp[i(nx + my) + \sigma t].$$

Thus equations (4.2.3) become

$$\sigma \xi = L\xi + DJ - \frac{1}{N}\xi + \sqrt{T}\ Dw,$$

$$\sigma P_m J = QD\xi + LJ,$$

$$\sigma Lw = L^2 w - a^2\sqrt{R}\ \theta + L(Db) - \varepsilon a^2(Db) - \frac{1}{N}Lw - \sqrt{T}\ D\xi, \tag{4.2.4}$$

$$\sigma P_m b = QDw + Lb - \varepsilon a^2 b,$$

$$\sigma P_r \theta = L\theta - H\sqrt{R}\ \ w.$$

Eliminating J from equation $(4.2.4)_1$ using equation $(4.2.4)_2$, thus

$$[(L - \sigma P_m)(L - \sigma - \frac{1}{N}) - QD^2]\xi = -\sqrt{T}(L - P_m\sigma)Dw. \tag{4.2.5}$$

We may eliminate b, θ and ξ from equation $(4.2.4)_3$ by applying the operator

$(L - \sigma P_m - \varepsilon a^2)(L - \sigma P_r)[(L - \sigma P_m)(L - \sigma - \frac{1}{N}) - QD^2]$ to equations $(4.2.4)_3$

and using equation (4.2.5) to obtain a twelfth order ordinary differential equation to be satisfied by w.

$$
\begin{aligned}
&\sigma^5 P_m P_r Lw - \sigma^4 \Big\{ P_m[P_m + 2P_r(1+P_m)]L^2 w - [\frac{2P_r P_m^2}{N} + 2P_r P_m \varepsilon a^2]Lw \Big\} \\
&+\sigma^3 \Big\{ [P_r(P_m+1)^2 + 2P_m(P_m+P_r+1)]L^3 w - [\frac{2P_m}{N}(P_m + P_r(2+P_m)) \\
&+\varepsilon a^2 (P_m + P_r + 2P_m P_r)]L^2 w - P_m P_r (2QD^2 - \frac{P_m}{N^2} - \frac{2\varepsilon a^2}{N})Lw - a^2 RHP_m^2 w \\
&+ TP_r P_m^2 D^2 w \Big\} - \sigma^2 \Big\{ [(P_m+1)^2 + 2(P_m + P_r + P_m P_r)]L^4 w \\
&-\frac{1}{N}[4P_m(1+P_r) + 2(P_r + P_m^2) - \varepsilon a^2 (1 + 2(P_m+P_r) + P_m P_r)]L^3 w \\
&+[\frac{P_m^2}{N^2}(1+\frac{2P_r}{P_m}) - 2QD^2(P_m + P_r + P_m P_r) + \frac{2\varepsilon a^2}{N}(P_r + P_m + P_r P_m)]L^2 w \\
&+P_m P_r[(\frac{2Q}{N} + T(2+\frac{P_m}{P_r}))D^2 - a^2 R\frac{H}{P_r}(2+P_m) + \varepsilon a^2 (Q(2P_r + P_m + P_r P_m)D^2 \\
&-\frac{P_r P_m}{N^2})]Lw + [a^2 \frac{R}{N} HP_m^2 - \varepsilon a^2 P_m P_r ((\frac{Q}{N} + T)D^2 - \frac{a^2}{P_r} RH)]w \Big\} \\
&+\sigma \Big\{ (2P_m + P_r + 2)L^5 w - [\frac{2}{N}(P_r + 2P_m + 1) + \varepsilon a^2 (P_r + P_m + 2)]L^4 w \\
&+[-(2QD^2(1+P_m+P_r) - \frac{1}{N^2}(2P_m + P_r)) + \frac{2\varepsilon a^2}{N}(1 + P_r + P_m)]L^3 w \\
&+[-a^2 RH(1+2P_m) + (\frac{2Q}{N}(P_m+P_r) + T(P_r + 2P_m))D^2 + \varepsilon a^2 (QD^2(2+2P_r+P_m) \\
&-\frac{1}{N^2}(P_r + P_m))]L^2 w + [(2a^2 \frac{R}{N} P_m H + Q^2 D^4 P_r) + \varepsilon a^2 (a^2 RH(1+P_m) \\
&-(\frac{Q}{N}(P_m + P_r) + T(P_r + P_m))D^2)]Lw + [a^2 RQHP_m D^2 - \varepsilon a^2 (a^2 P_m \frac{R}{N} H \\
&+P_r Q^2 D^4)]w \Big\} - L^6 w + (\frac{2}{N} + \varepsilon a^2)L^5 w + (2QD^2 - \frac{1}{N^2} - \frac{2\varepsilon a^2}{N})L^4 w \\
&+[a^2 RH - (2\frac{Q}{N} + T)D^2 + \varepsilon a^2 (\frac{1}{N} - 2QD^2)]L^3 w + [-(a^2 \frac{R}{N} H + Q^2 D^4) \\
&+\varepsilon a^2 ((\frac{2Q}{N} + T)D^2 - a^2 RH)]L^2 w + [\varepsilon a^2 (Q^2 D^4 + \frac{a^2 R}{N} H) - a^2 RQHD^2]Lw \\
&+\varepsilon a^4 RHQD^2 = 0.
\end{aligned}
\tag{4.2.6}
$$

Now we shall consider both boundaries to be free but later on we shall present results for the corresponding rigid boundary value problems. For the free boundary value problems

$$w = D^2 w = 0 \qquad \text{on} \qquad z = 0, 1$$

thus equation (4.2.6) has eigenfunctions $w = A\sin(l\pi z)$ where A is constant and l is an integer. Consequently $Lw = -\lambda w$ where $\lambda = l^2\pi^2 + a^2$ and σ satisfies the fifth order equation,

$$\sigma^5 P_m P_r + \sigma^4 \left\{ P_m[P_m + 2P_r(1+P_m)]\lambda - [\frac{2P_r P_m^2}{N} + 2P_r P_m \varepsilon a^2] \right\}$$

$$+\sigma^3 \left\{ [P_r(P_m+1)^2 + 2P_m(P_m + P_r + 1)]\lambda^2 + [\frac{2P_m}{N}(P_m + P_r(2+P_m)) \right.$$

$$+\varepsilon a^2 (P_m + P_r + 2P_m P_r)]\lambda + P_m P_r (2Ql^2\pi^2 + \frac{P_m}{N^2} + \frac{2\varepsilon a^2}{N}) + P^2(a^2 RH$$

$$\left. + TP_r l^2\pi^2)\lambda^{-1} \right\} + \sigma^2 \left\{ [(P_m+1)^2 + 2(P_m + P_r + P_m P_r)]\lambda^3 \right.$$

$$+\frac{1}{N}[4P_m(1+P_r) + 2(P_r + P_m^2) + \varepsilon a^2(1 + 2(P_m + P_r) + P_m P_r)]\lambda^2$$

$$+[\frac{P_m^2}{N^2}(1 + \frac{2P_r}{P_m}) + 2Ql^2\pi^2(P_m + P_r + P_m P_r) + \frac{2\varepsilon a^2}{N}(P_r + P_m + P_r P_m)]\lambda$$

$$+ P_m P_r[(\frac{2Q}{N} + T(2 + \frac{P_m}{P_r}))l^2\pi^2 + a^2 R\frac{H}{P_r}(2+P_m)] + \varepsilon a^2(Q(2P_r + P_m + P_r P_m)l^2\pi^2$$

$$\left. + \frac{P_r P_m}{N^2})] + [a^2\frac{R}{N}HP_m^2 + \varepsilon a^2 P_m P_r((\frac{Q}{N} + T)l^2\pi^2 + \frac{a^2}{P_r}RH)]\lambda^{-1} \right\}$$

$$+\sigma \left\{ (2P_m + P_r + 2)\lambda^4 + [\frac{2}{N}(P_r + 2P_m + 1) + \varepsilon a^2(P_r + P_m + 2)]\lambda^3 \right.$$

$$+[(2Ql^2\pi^2(1 + P_m + P_r) + \frac{1}{N^2}(2P_m + P_r)) + \frac{2\varepsilon a^2}{N}(1 + P_r + P_m)]\lambda^2$$

$$+[a^2RH(1+2P_m)+(\frac{2Q}{N}(P_m+P_r)+T(P_r+2P_m))l^2\pi^2+\varepsilon a^2(Ql^2\pi^2(2+2P_r+P_m)$$
$$+\frac{1}{N^2}(P_r+P_m))]\lambda+(2a^2\frac{R}{N}P_mH+Q^2l^4\pi^4P_r)+\varepsilon a^2(a^2RH(1+P_m)$$
$$+(\frac{Q}{N}(P_m+P_r)+T(P_r+P_m))l^2\pi^2)+[a^2RQHP_ml^2\pi^2+\varepsilon a^2(a^2P_m\frac{R}{N}H$$
$$+P_rQ^2l^4\pi^4)]\lambda^{-1}\Bigg\}+(\lambda+\varepsilon a^2)\Bigg[\lambda^4+\frac{2}{N}\lambda^3+(2Ql^2\pi^2+\frac{1}{N^2})\lambda^2$$
$$+[a^2RH+(\frac{2Q}{N}+T)l^2\pi^2]\lambda+(a^2\frac{R}{N}H+Q^2l^4\pi^4)+a^2RHQl^2\pi^2\lambda^{-1}\Bigg]=0. \tag{4.2.7}$$

Since the coefficients of this polynomial are real, then its solutions are one of the following :

1. All solutions are real.
2. Three solutions are real and two are complex conjugate pair solutions.
3. One solution is real and four are complex conjugate solutions.

The stationary instability happens if any real solution is positive while overstability happens if any real part of the complex conjugate solutions is positive . Solutions of (4.2.7) are functions of $P_r, P_m, N, \varepsilon, Q, T$ and R and we have to examine how the nature of these solutions depends on $P_r, P_m, N, \varepsilon, Q, T$ and R in the context of heating the fluid layer from below.

Let us assume that $|\underline{B}|$ is an increasing function of $|\underline{H}|$ so that

$\frac{dB}{dH}>0$. Consequently $\frac{d(\phi B)}{dB}>0 \Rightarrow \phi+B\phi_B>0$

$\therefore 1+\frac{B\phi_B}{\phi}>0 \Rightarrow 1+\varepsilon>0$ and so $\varepsilon>-1$. Since $\varepsilon>-1$ then

$$\lambda + \varepsilon a^2 > 0. \tag{4.2.8}$$

Stationary convection case

To find the critical Rayleigh number for the onset of stationary convection we set $\sigma = 0$ in equation (4.2.7). Thus

$$(\lambda + \varepsilon a^2)\left[\frac{\lambda}{a^2 C}(C^2 + T\lambda l^2\pi^2) - R \right] = 0.$$

where $C = \lambda^2 + \dfrac{\lambda}{N} + l^2\pi^2 Q$.

i.e.

$$R = \frac{\lambda}{a^2 C}(C^2 + T\lambda l^2\pi^2). \tag{4.2.9}$$

Since this equation does not contain P_r, P_m or ε then the critical Rayleigh number for stationary convection is independent of P_r, P_m or ε, which means that the non-linear relation between $\underline{B}$ and $\underline{H}$ has no effect on the development of stationary instability. From equation (4.2.9) we find that

$$\begin{aligned} \frac{\mathrm{d}R}{\mathrm{d}Q} &= \frac{\lambda l^2\pi^2}{a^2}\left[1 - \frac{T\lambda l^2\pi^2}{C^2}\right], \\ \frac{\mathrm{d}R}{\mathrm{d}N} &= -\frac{\lambda^2}{a^2 N^2}\left[1 - \frac{T l^2\pi^2\lambda}{C^2}\right], \\ \frac{\mathrm{d}R}{\mathrm{d}T} &= \frac{\lambda^2 l^2\pi^2}{a^2 C^2}. \end{aligned} \tag{4.2.10}$$

It is clear from equation $(4.2.10)_1$ that the magnetic field has a stabilizing effect on the system in the absence of rotation. Also it has a stabilizing effect on the system in the presence of rotation provided that

$$T < \frac{C^2}{\lambda l^2 \pi^2} \; .$$

From equation $(4.2.10)_2$ we find that the permeability of porous medium has a destabilizing effect on the system in the absence of rotation. Also it has a destabilizing effect on the system in the presence of rotation provided that

$$T < \frac{C^2}{l^2 \pi^2 \lambda} \; .$$

From equation $(4.2.10)_3$ it is clear that the rotation has a stabilizing effect on the system.

The overstability case

Since the equations are so complicated, we were unable to obtain analytical solution for the overstability case but we have produced numerical solutions for the corresponding problem.

4.3 Numerical discussion

The eigenvalue problem (4.2.4) together with the boundary conditions are solved using expansion of Chebyshev polynomials. The non-linear relationship between the magnetic field $\underline{H}$ and the magnetic induction $\underline{B}$ has no effects on the relation between the critical Rayleigh number R and the magnetic parameter Q for the stationary convection case for different boundary conditions. However it has a great effect in the development of instabilities through overstability case. The relation between the critical

Rayleigh number R and the magnetic parameter Q for the overstability case for different values of the porous medium permeability when both boundaries are free is displayed in figures (25) and (26) when $T=10^4$ for $\varepsilon=1, 2$ respectively and in figures (27), (28) and (29) when $T=5\times10^4$ for $\varepsilon=1,2,3$ respectively. For rigid boundary conditions the relation is displayed in figures (30) and (31) when $T=10^4$ for $\varepsilon=0.25,0.5$ respectively and in figures (32) and (33) when $T=5\times10^4$ for $\varepsilon=0.25,0.5$ respectively. The figures show that as Q increases R increases which means that the magnetic field has a stabilizing effect. Also it appears from the figures that as the permeability of the porous medium, N, decreases R increases which means that as the fluid becomes less porous it becomes more stable. Moreover the critical value of the magnetic parameter Q, at which overstability becomes possible, increases as the porous medium permeability N decreases. The numerical results are listed in tables (24)-(28) for free boundary conditions and in tables (29)-(32) for rigid boundary conditions. Figure (34) shows a comparison between the free boundary conditions and the rigid boundary conditions for $T=10^4$ when $\varepsilon=0.5$. The numerical results are listed in table (33).

In fact, it appears that as the value of the parameter ε increases, the critical Rayleigh number increase which means that the non-linearity has a stabilizing effect for the overstability case. The fortran codes for the free and rigid boundary problems are listed in appendices (IV) and (V) respectively.

Table 24. The relation between R and Q for the overstability case when both boundaries are free for $T = 10000$ and $\varepsilon = 1$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
200	-	-	-	-	-	-
400	3.103	7008.053	3.221	11136.060	-	-
800	3.164	8120.613	3.371	13363.901	-	-
1200	3.362	9755.839	3.530	15644.370	-	-
1600	3.556	11531.843	3.681	17926.803	3.175	67674.443
2000	3.732	13351.636	3.820	20195.019	3.220	70554.605
2400	3.889	15183.086	3.948	22443.970	3.264	73409.711
2800	4.029	17013.885	4.066	24672.654	3.306	76241.551
3200	4.156	18839.028	4.175	26881.603	3.347	79051.749
3600	4.271	20656.537	4.277	29071.910	3.386	81841.767
4000	4.376	22465.783	4.373	31244.830	3.423	84612.913
4500	4.497	24715.735	4.484	33938.427	3.469	88052.083
5000	4.606	26953.348	4.588	36609.077	3.513	91465.637
5500	4.707	29179.438	4.686	39258.813	3.555	94855.379
6000	4.800	31394.878	4.777	41889.430	3.596	98222.912
6500	4.886	33600.506	4.864	44502.502	3.636	101569.664
7000	4.967	35797.100	4.946	47099.411	3.674	104896.918
7500	5.042	37985.364	5.024	49681.373	3.711	108205.828
8000	5.113	40165.932	5.099	52249.462	3.747	111497.438
8500	5.180	42339.369	5.170	54804.633	3.782	114772.696
9000	5.244	44506.183	5.238	57347.732	3.816	118032.466
9500	5.305	46666.827	5.304	59879.521	3.850	121277.536
10000	5.362	48821.710	5.367	62400.680	3.882	124508.631

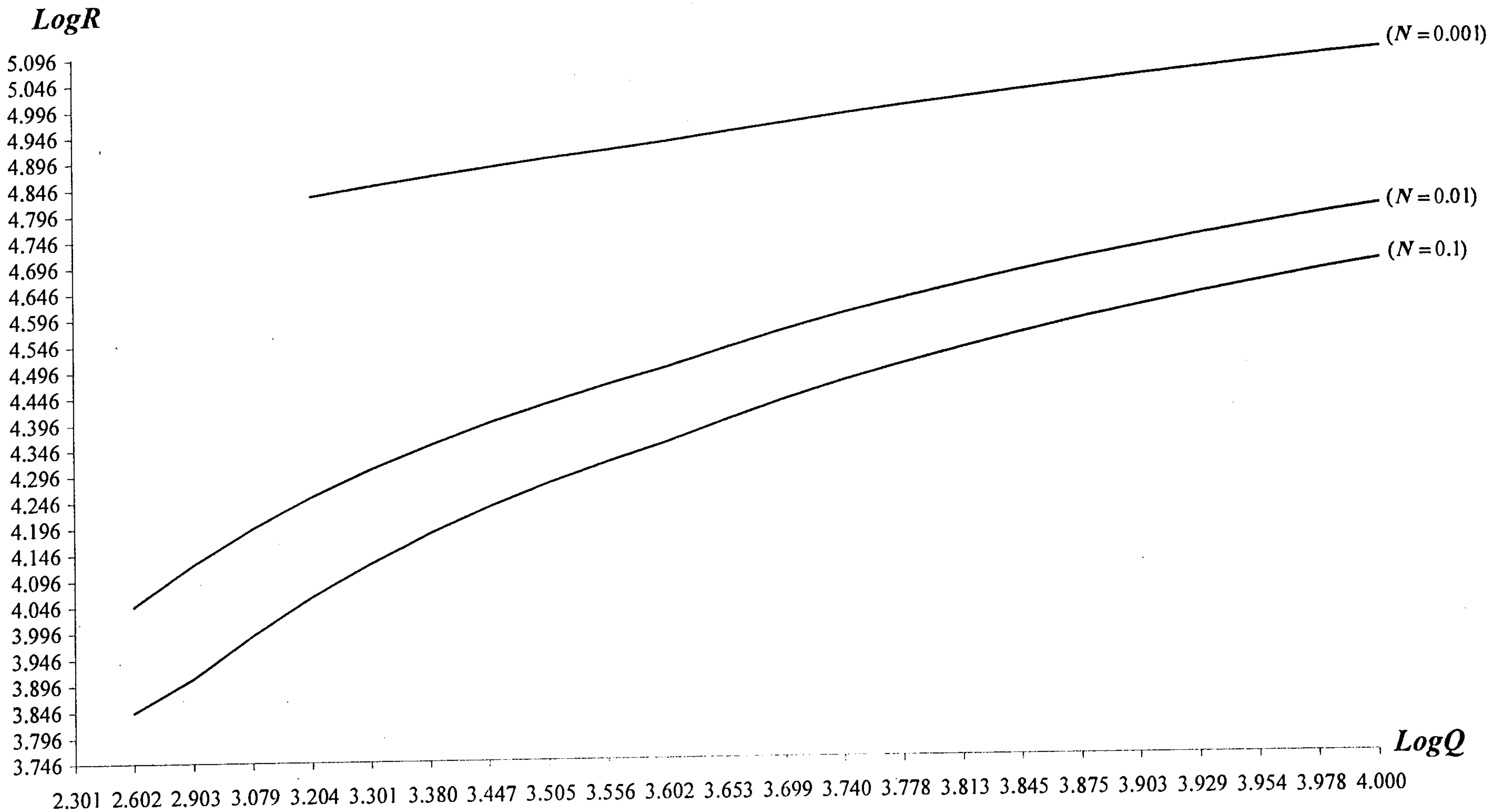

Figure 25. The relation between R and Q for the overstability case when both boundaries are free for $T = 10000$ and $\varepsilon = 1$.

Table 25. The relation between R and Q for the overstability case when both boundaries are free for $T = 10000$ and $\varepsilon = 2$.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
200	-	-	-	-	-	-
400	-	-	-	-	-	-
800	2.964	10710.972	3.215	16324.527	-	-
1200	3.130	13277.343	3.359	19623.631	-	-
1600	3.300	16028.664	3.496	22933.001	-	-
2000	3.455	18845.620	3.622	26233.042	-	-
2400	3.593	21686.901	3.738	29517.069	-	-
2800	3.717	24535.758	3.845	32783.167	-	-
3200	3.828	27384.762	3.944	36031.313	-	-
3600	3.929	30230.482	4.035	39262.244	3.252	96266.846
4000	4.021	33071.354	4.121	42476.973	3.288	100092.369
4500	4.126	36614.695	4.221	46474.241	3.332	104847.832
5000	4.221	40149.194	4.314	50449.911	3.374	109575.920
5500	4.308	43675.075	4.401	54405.848	3.414	114278.553
6000	4.388	47192.777	4.482	58343.721	3.453	118957.441
6500	4.462	50702.815	4.559	62265.007	3.491	123614.114
7000	4.531	54205.712	4.632	66171.014	3.527	128249.945
7500	4.595	57701.970	4.701	70062.895	3.563	132866.171
8000	4.656	61192.058	4.767	73941.673	3.597	137463.914
8500	4.713	64676.406	4.830	77808.257	3.630	142044.190
9000	4.767	68155.405	4.890	81663.458	3.663	146607.925
9500	4.819	71629.411	4.948	85508.002	3.695	151155.968
10000	4.867	75098.746	5.003	89342.542	3.726	155689.096

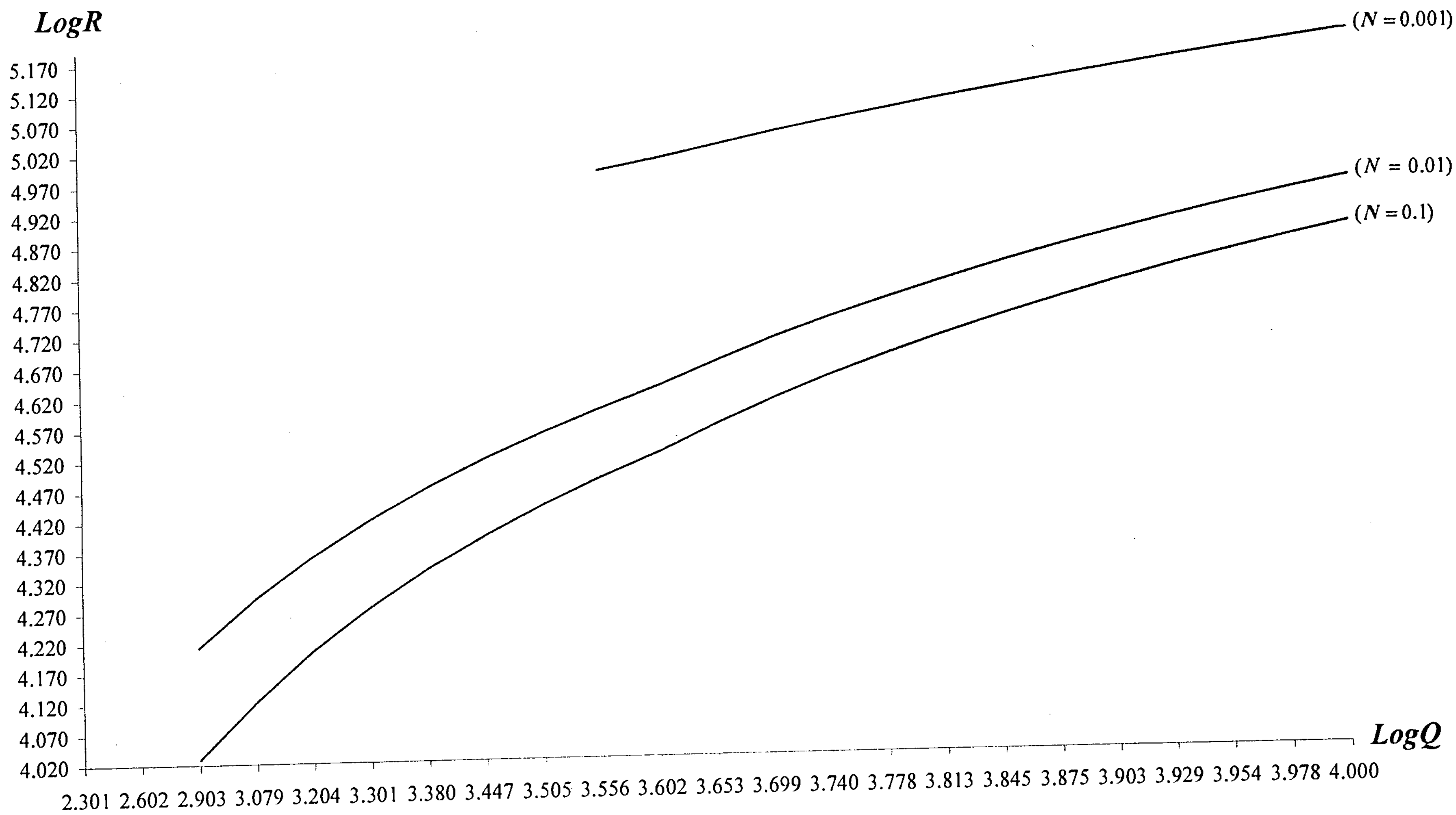

Figure 26. The relation between R and Q for the overstability case when both boundaries are free for $T = 10000$ and $\varepsilon = 2$.

Table 26. The relation between R and Q for the overstability case when both boundaries are free for T =50000 and $\varepsilon = 1$.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
200	-	-	-	-	-	-
400	-	-	-	-	-	-
800	3.087	13505.174	3.422	17359.969	-	-
1200	3.098	13365.040	3.441	18732.868	-	-
1600	3.193	14148.833	3.518	20404.094	3.188	68556.801
2000	3.308	15344.451	3.614	22238.397	3.232	71384.991
2400	3.427	16753.848	3.717	24165.429	3.274	74193.793
2800	3.542	18285.694	3.819	26146.797	3.314	76983.845
3200	3.654	19892.404	3.919	28160.408	3.353	79756.027
3600	3.759	21546.919	4.015	30192.955	3.390	82511.250
4000	3.860	23232.824	4.108	32236.125	3.427	85250.394
4500	3.979	25368.771	4.218	34797.063	3.472	88652.966
5000	4.090	27524.635	4.323	37359.771	3.514	92033.198
5500	4.195	29692.289	4.422	39920.260	3.555	95392.450
6000	4.294	31866.494	4.516	42476.091	3.596	98731.962
6500	4.388	34043.786	4.606	45025.784	3.634	102052.860
7000	4.477	36221.834	4.692	47568.457	3.672	105356.174
7500	4.561	38399.054	4.774	50103.619	3.708	108642.845
8000	4.642	40574.365	4.852	52631.028	3.744	111913.734
8500	4.719	42747.031	4.927	55150.608	3.779	115169.632
9000	4.792	44916.558	4.999	57662.394	3.812	118411.267
9500	4.863	47082.621	5.069	60166.491	3.845	121639.312
10000	4.930	49245.020	5.136	62663.050	3.877	124854.386

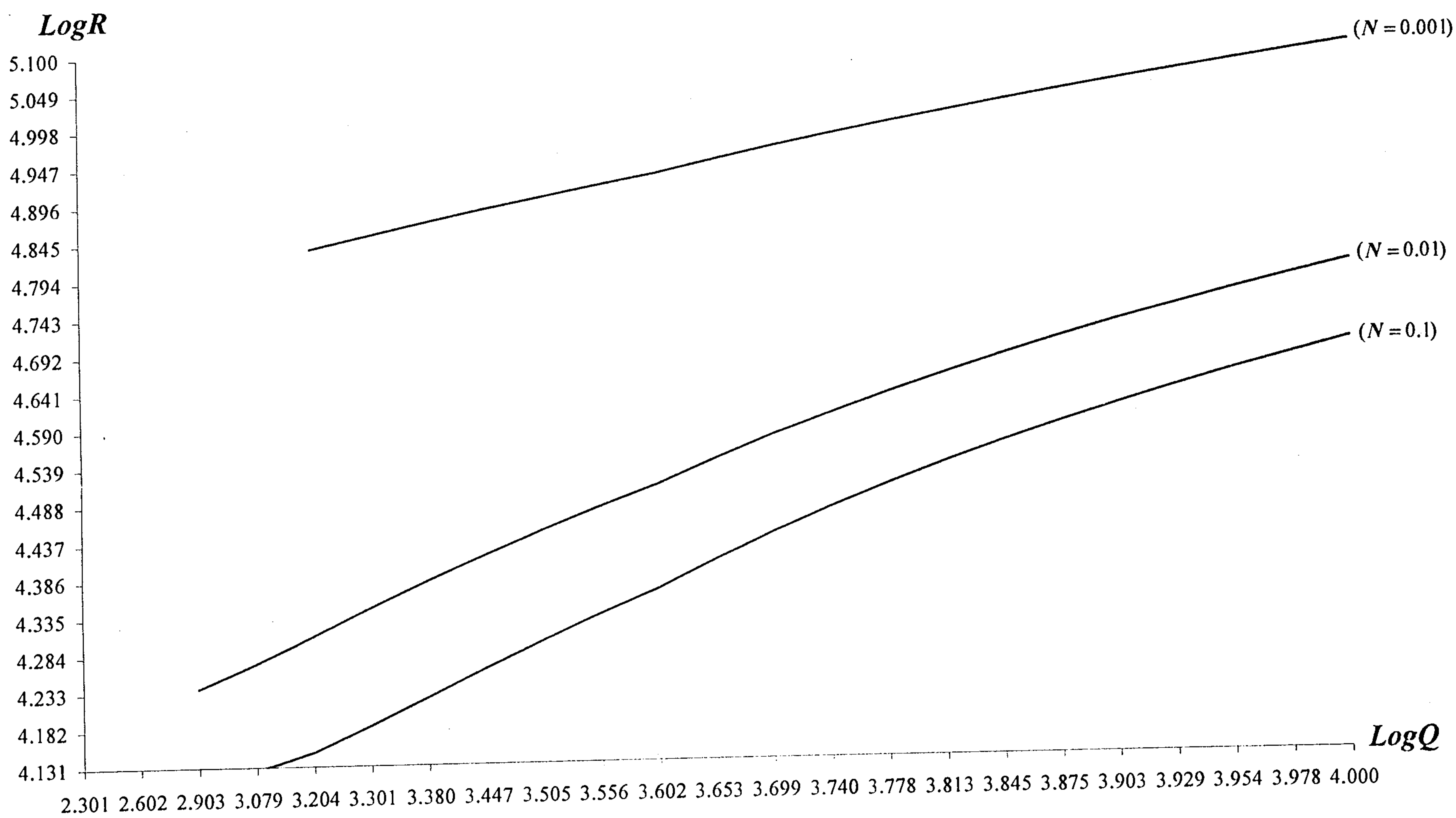

Figure 27. The relation between R and Q for the overstability case when both boundaries are free for $T = 50000$ and $\varepsilon = 1$.

Table 27. The relation between R and Q for the overstability case when both boundaries are free for $T = 50000$ and $\varepsilon = 2$.

	(*N*=0.1)		(*N*=0.01)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
200	-	-	-	-	-	-
400	-	-	-	-	-	-
800	-	-	-	-	-	-
1200	2.846	17771.450	3.282	23337.646	-	-
1600	2.919	19289.038	3.343	25936.805	-	-
2000	3.016	21323.716	3.426	28727.417	-	-
2400	3.119	23631.376	3.517	31630.493	-	-
2800	3.220	26098.206	3.609	34601.888	-	-
3200	3.318	28664.349	3.699	37615.730	-	-
3600	3.412	31295.490	3.786	40656.180	-	-
4000	3.501	33970.634	3.869	43713.188	-	-
4500	3.607	37356.339	3.969	47547.936	3.334	105559.122
5000	3.706	40773.038	4.064	51390.153	3.375	110249.385
5500	3.800	44209.992	4.153	55234.686	3.414	114917.498
6000	3.888	47660.165	4.238	59078.266	3.452	119564.718
6500	3.972	51118.811	4.319	62918.801	3.489	124192.208
7000	4.051	54582.663	4.396	66754.951	3.525	128801.043
7500	4.127	58049.418	4.469	70585.866	3.560	133392.209
8000	4.198	61517.491	4.539	74411.029	3.594	137966.615
8500	4.266	64985.669	4.606	78230.138	3.627	142525.097
9000	4.332	68453.162	4.671	82043.048	3.659	147068.427
9500	4.394	71919.358	4.732	85849.714	3.690	151597.320
10000	4.454	75383.831	4.792	89650.161	3.721	156112.434

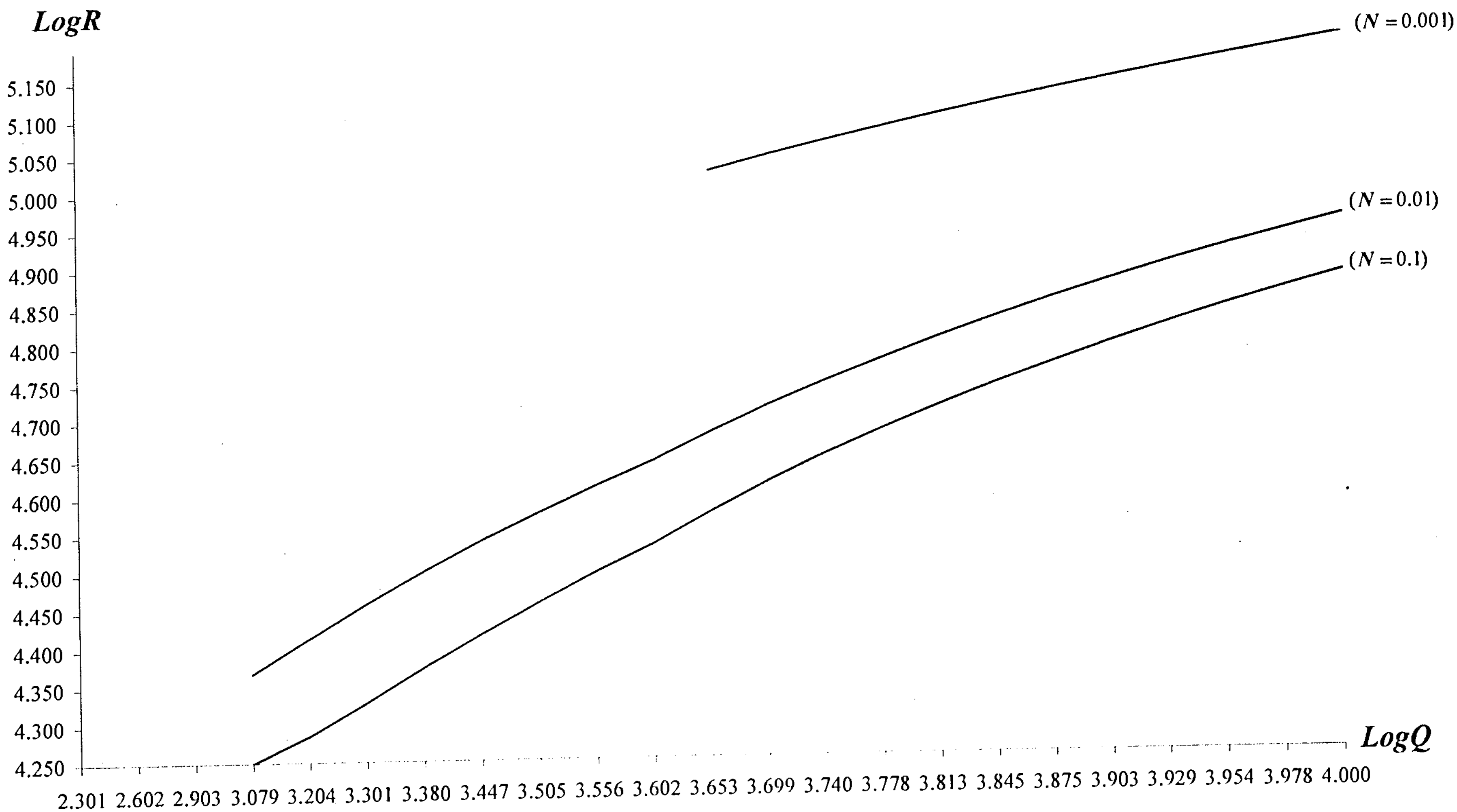

Figure 28. The relation between R and Q for the overstability case when both boundaries are free for T=50000 and $\varepsilon = 2$.

Table 28. The relation between R and Q for the overstability case when both boundaries are free for $T = 50000$ and $\varepsilon = 3$

	(*N*=0.1)		(*N*=0.01)		(*N*=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
2000	-	-	-	-	-	-
4000	3.149	45556.151	3.652	55568.642	-	-
6000	3.463	64839.629	3.969	76369.415	-	-
8000	3.708	84426.080	4.223	97220.830	-	-
10000	3.904	104101.778	4.433	118032.031	-	-
12000	4.065	123804.000	4.611	138785.101	-	-
14000	4.200	143510.784	4.765	159479.824	-	-
16000	4.315	163213.847	4.901	180120.599	-	-
18000	4.415	182910.115	5.023	200712.728	-	-
20000	4.502	202598.650	5.133	221261.283	-	-
22000	4.580	222279.420	5.233	241770.801	-	
24000	4.650	241952.766	5.326	262245.237	-	-
26000	4.713	261619.172	5.411	282688.012	-	-
28000	4.771	281279.157	5.491	303102.085	-	-
30000	4.824	300933.224	5.566	323490.019	4.399	412714.811
32000	4.873	320581.843	5.636	343854.045	4.458	434754.590
34000	4.918	340225.444	5.702	364196.113	4.515	456733.303
36000	4.960	359864.414	5.765	384517.936	4.570	478655.467
38000	4.999	379499.103	5.824	404821.028	4.622	500525.066
40000	5.037	399129.824	5.881	425106.730	4.673	522345.644

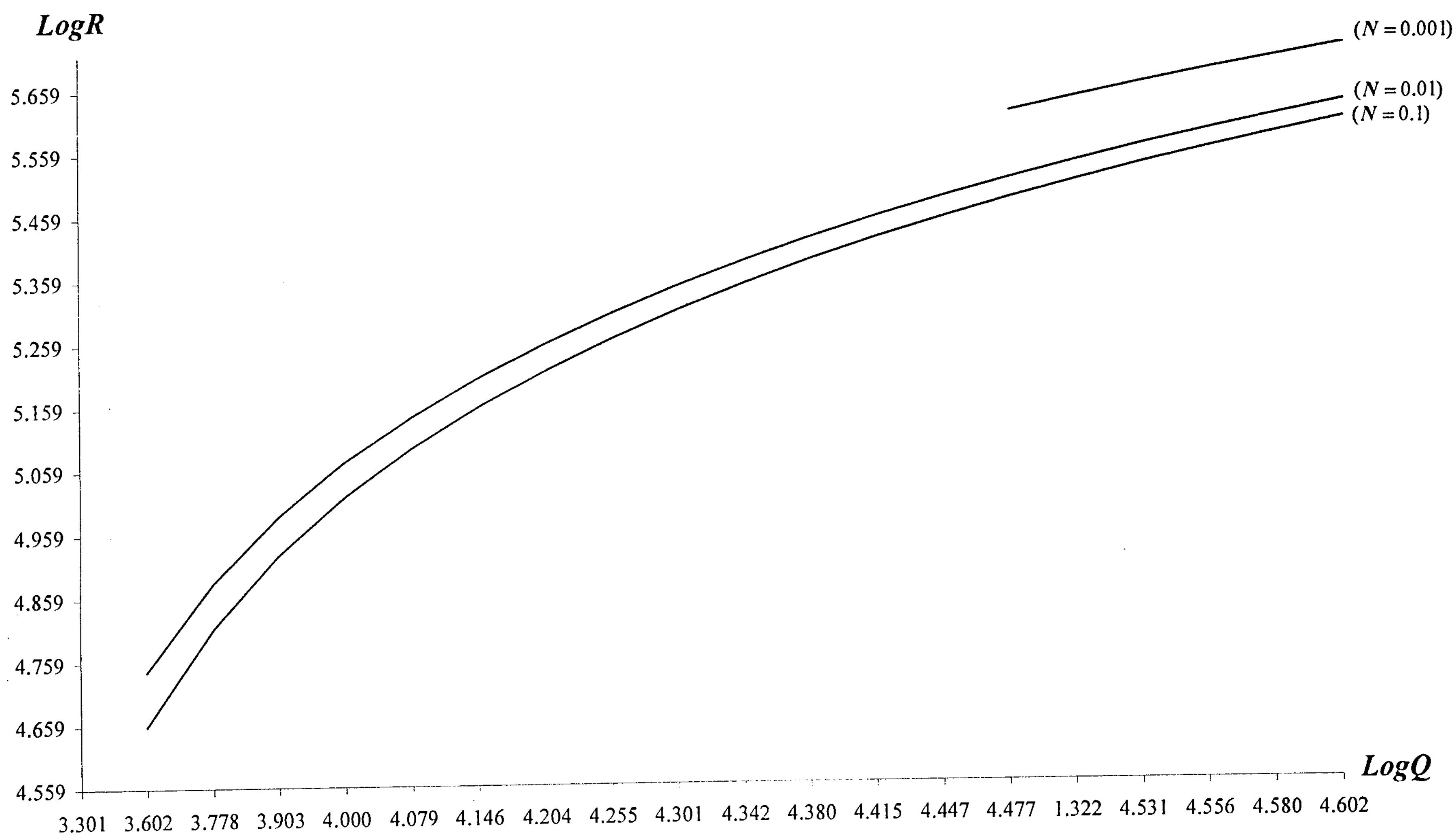

Figure 29. The relation between R and Q for the overstability case when both boundaries are free for T=50000 and $\varepsilon = 3$.

Table 29. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 10000$ and $\varepsilon = 0.25$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
500	4.732	10954.107	-	-	-	-
1000	5.246	14369.954	4.893	22173.590	-	-
1500	5.621	17576.588	5.251	26158.627	-	-
2000	5.915	20603.294	5.540	29863.210	-	-
2500	6.157	23491.403	5.783	33364.693	-	-
3000	6.363	26270.626	5.993	36710.440	-	-
3500	6.544	28961.850	6.179	39931.804	-	-
4000	6.704	31580.159	6.346	43050.792	-	-
4500	6.849	34136.796	6.498	46083.552	-	-
5000	6.981	36640.379	6.637	49042.344	-	-
5500	7.103	39097.678	6.766	51936.740	-	-
6000	7.216	41514.120	6.887	54774.383	-	-
6500	7.322	43894.135	6.999	57561.492	5.167	143175.284
7000	7.421	46241.395	7.105	60303.210	5.242	147459.986
7500	7.514	48558.979	7.205	63003.847	5.314	151669.804
8000	7.603	50849.507	7.300	65667.058	5.382	155811.056
8500	7.686	53115.223	7.391	68295.978	5.449	159889.199
9000	7.766	55358.071	7.477	70893.316	5.513	163908.983
9500	7.843	57579.747	7.559	73461.436	5.574	167874.576
10000	7.916	59781.743	7.638	76002.414	5.634	171789.661

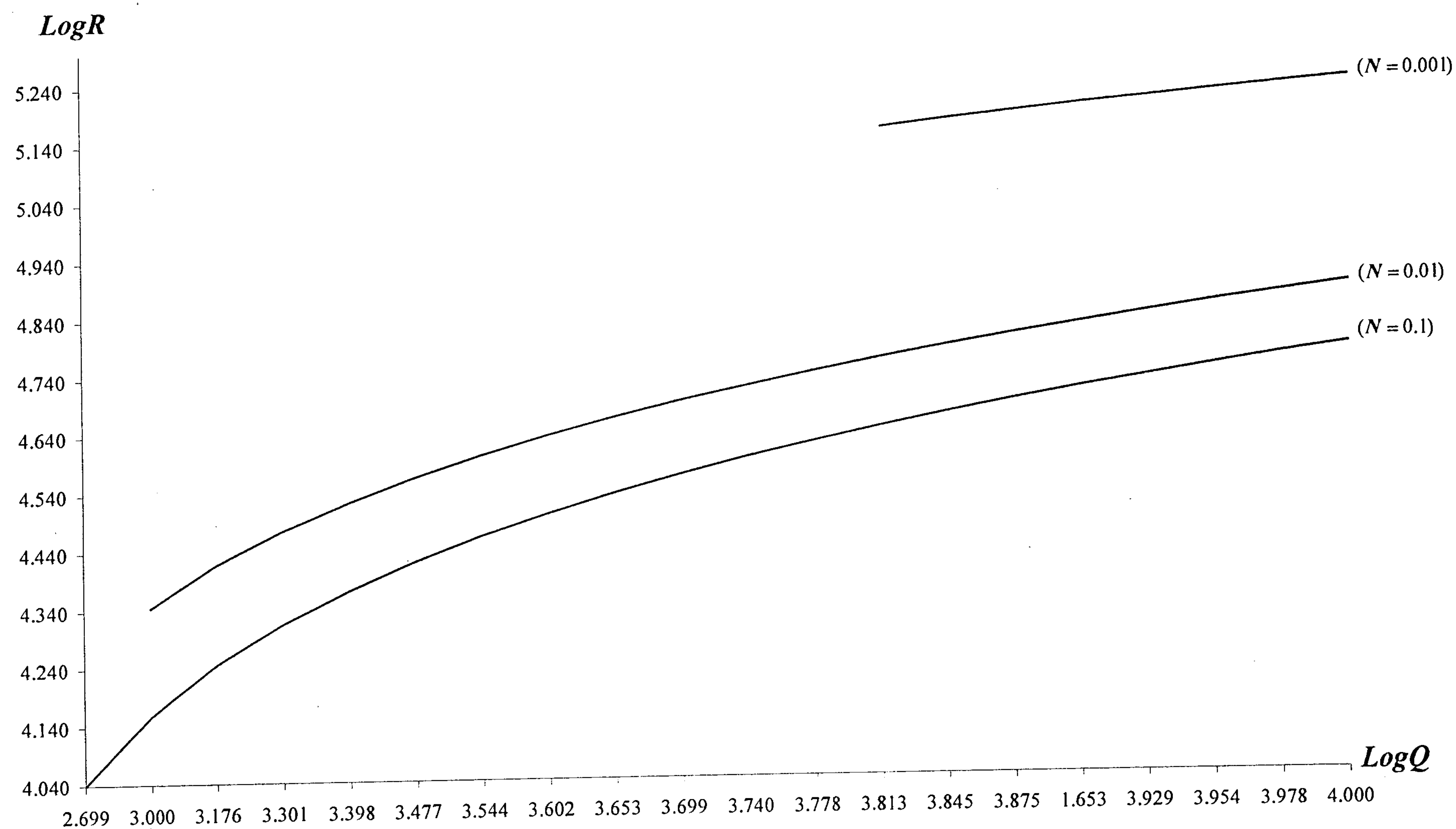

Figure 30. The relation between R and Q for the overstability case when both boundaries are rigid for T=10000 and $\varepsilon = 0.25$.

Table 30. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 10000$ and $\varepsilon = 0.5$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	a	R	a	R	a	R
500	-	-	-	-	-	-
1000	5.184	15600.673	-	-	-	-
1500	5.559	19266.817	5.184	27978.459	-	-
2000	5.853	22739.607	5.460	32139.890	-	-
2500	6.095	26063.724	5.712	36088.562	-	-
3000	6.306	29271.102	5.921	39874.203	-	-
3500	6.482	32384.035	6.106	43529.319	-	-
4000	6.643	35418.760	6.273	47076.865	-	-
4500	6.788	38387.286	6.421	50533.722	-	-
5000	6.920	41298.839	6.563	53912.652	-	-
5500	7.042	44160.668	6.691	57223.735	-	-
6000	7.155	46978.586	6.812	60474.946	-	-
6500	7.261	49757.340	6.922	63672.816	-	-
7000	7.360	52500.862	7.028	66822.733	-	-
7500	7.454	55212.454	7.129	69929.218	-	-
8000	7.543	57899.326	7.226	72996.127	5.317	164950.220
8500	7.626	60550.678	7.314	76026.751	5.382	169465.660
9000	7.706	63181.801	7.399	79023.909	5.446	173917.555
9500	7.783	65790.114	7.484	81990.129	5.507	178316.274
10000	7.856	68377.217	7.562	84927.572	5.567	182662.773

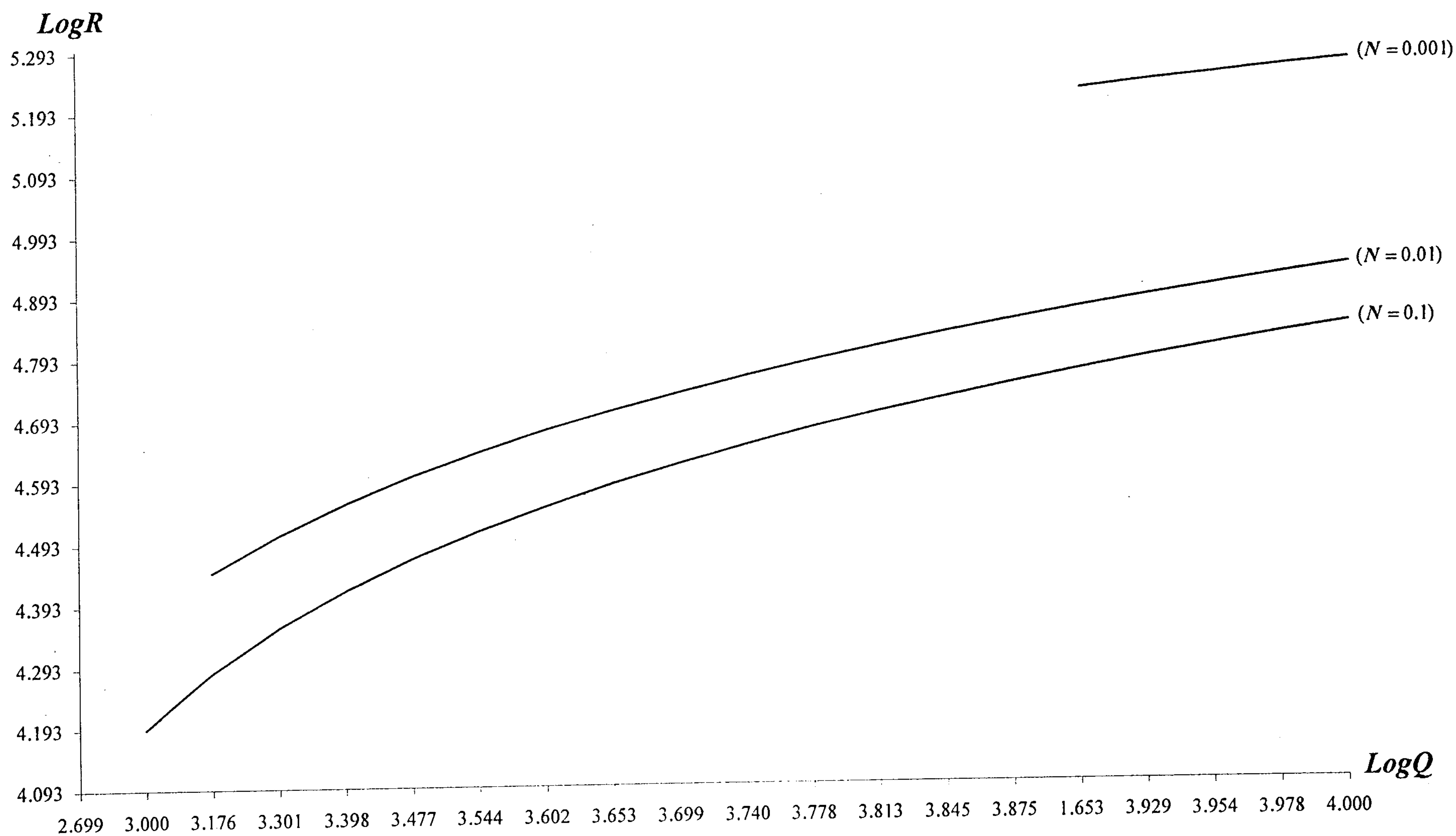

Figure 31. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 10000$ and $\varepsilon = 0.5$.

Table 31. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 50000$ and $\varepsilon = 0.25$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
500	-	-	-	-	-	-
1000	5.273	16891.951	-		-	-
1500	5.591	19396.916	5.250	27599.850	-	-
2000	5.867	22022.312	5.520	30991.202	-	-
2500	6.103	24649.786	5.755	34284.098	-	-
3000	6.309	27245.576	5.961	37481.167	-	-
3500	6.491	29800.414	6.145	40591.342	-	-
4000	6.653	32313.203	6.312	43624.153	-	-
4500	6.800	34785.703	6.464	46588.244	-	-
5000	6.934	37220.597	6.604	49491.090	-	-
5500	7.058	39620.741	6.734	52339.067	-	-
6000	7.173	41988.878	6.854	55137.601	-	-
6500	7.281	44327.529	6.968	57891.323	5.167	143451.222
7000	7.382	46638.970	7.074	60604.212	5.242	147719.474
7500	7.476	48925.242	7.175	63279.703	5.313	151916.105
8000	7.566	51188.167	7.271	65920.783	5.382	156044.516
8500	7.651	53429.370	7.362	68530.066	5.448	160110.881
9000	7.733	55650.309	7.449	71109.852	5.512	164119.801
9500	7.810	57852.290	7.532	73662.176	5.573	168075.321
10000	7.885	60036.489	7.612	76188.851	5.633	171981.021

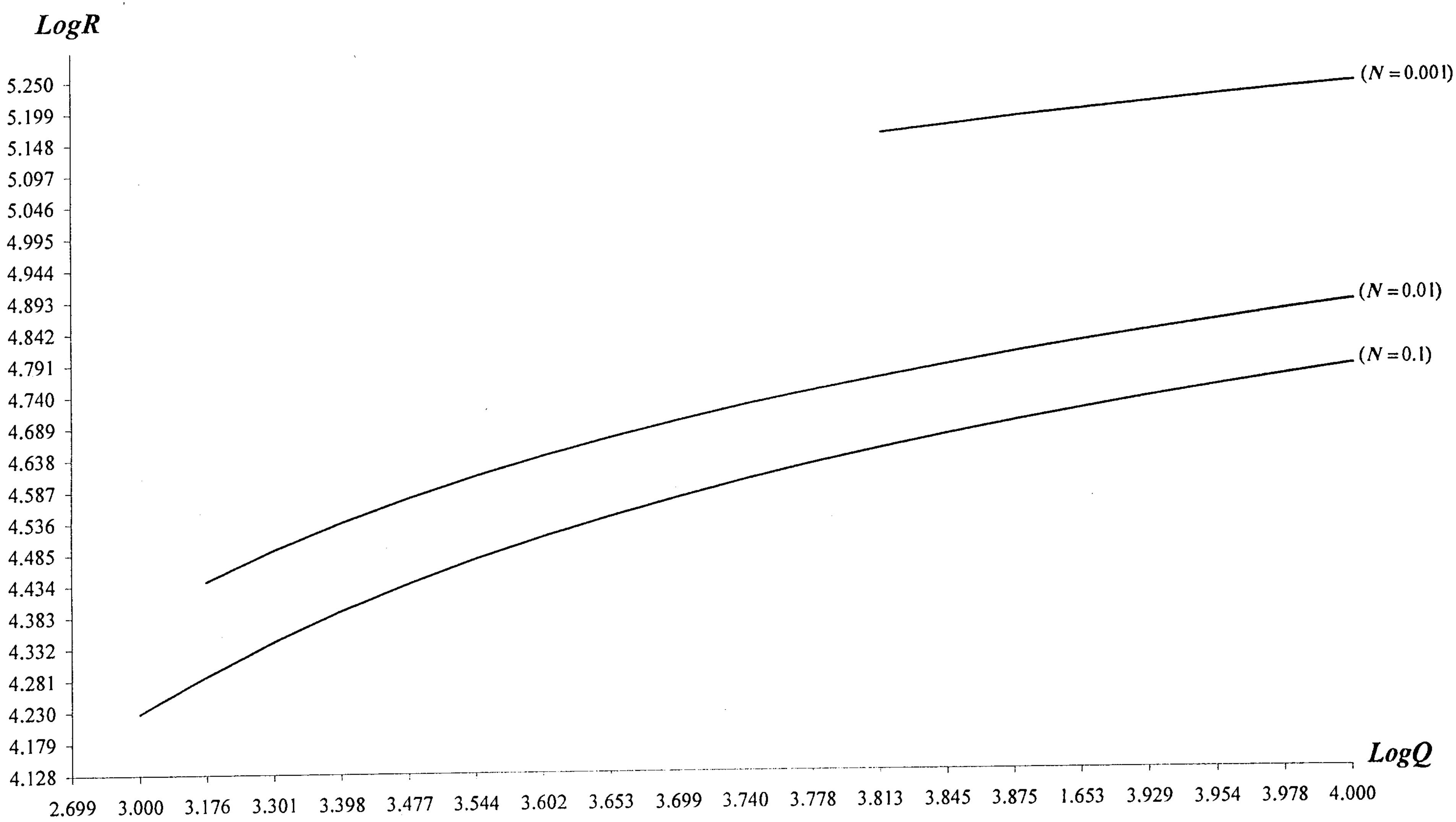

Figure 32. The relation between R and Q for the overstability case when both boundaries are rigid for T=50000 and $\varepsilon = 0.25$.

Table 32. The relation between R and Q for the overstability case when both boundaries are rigid for $T = 50000$ and $\varepsilon = 0.5$.

	(N=0.1)		(N=0.01)		(N=0.001)	
Q	***a***	***R***	***a***	***R***	***a***	***R***
500	-	-	-	-	-	-
1000	5.213	18262.542	-	-	-	-
1500	5.532	21195.997	-	-	-	-
2000	5.807	24249.124	5.454	33345.197	-	-
2500	6.049	27300.431	5.687	37076.688	-	-
3000	6.249	30315.532	5.886	40707.124	-	-
3500	6.431	33285.439	6.075	44245.844	-	-
4000	6.593	36209.374	6.241	47703.060	-	-
4500	6.740	39089.469	6.392	51087.804	-	-
5000	6.874	41928.750	6.531	54407.930	-	-
5500	6.998	44730.378	6.660	57670.131	-	-
6000	7.115	47497.362	6.781	60880.106	-	-
6500	7.222	50232.450	6.894	64042.720	-	-
7000	7.322	52938.122	7.000	67162.153	-	-
7500	7.416	55616.589	7.101	70242.015	-	-
8000	7.507	58269.823	7.196	73285.450	-	-
8500	7.592	60899.593	7.287	76295.205	5.383	169700.646
9000	7.674	63507.464	7.374	79273.702	5.446	174143.817
9500	7.752	66094.852	7.457	82223.084	5.507	178531.888
10000	7.826	68663.028	7.537	85145.259	5.565	182872.174

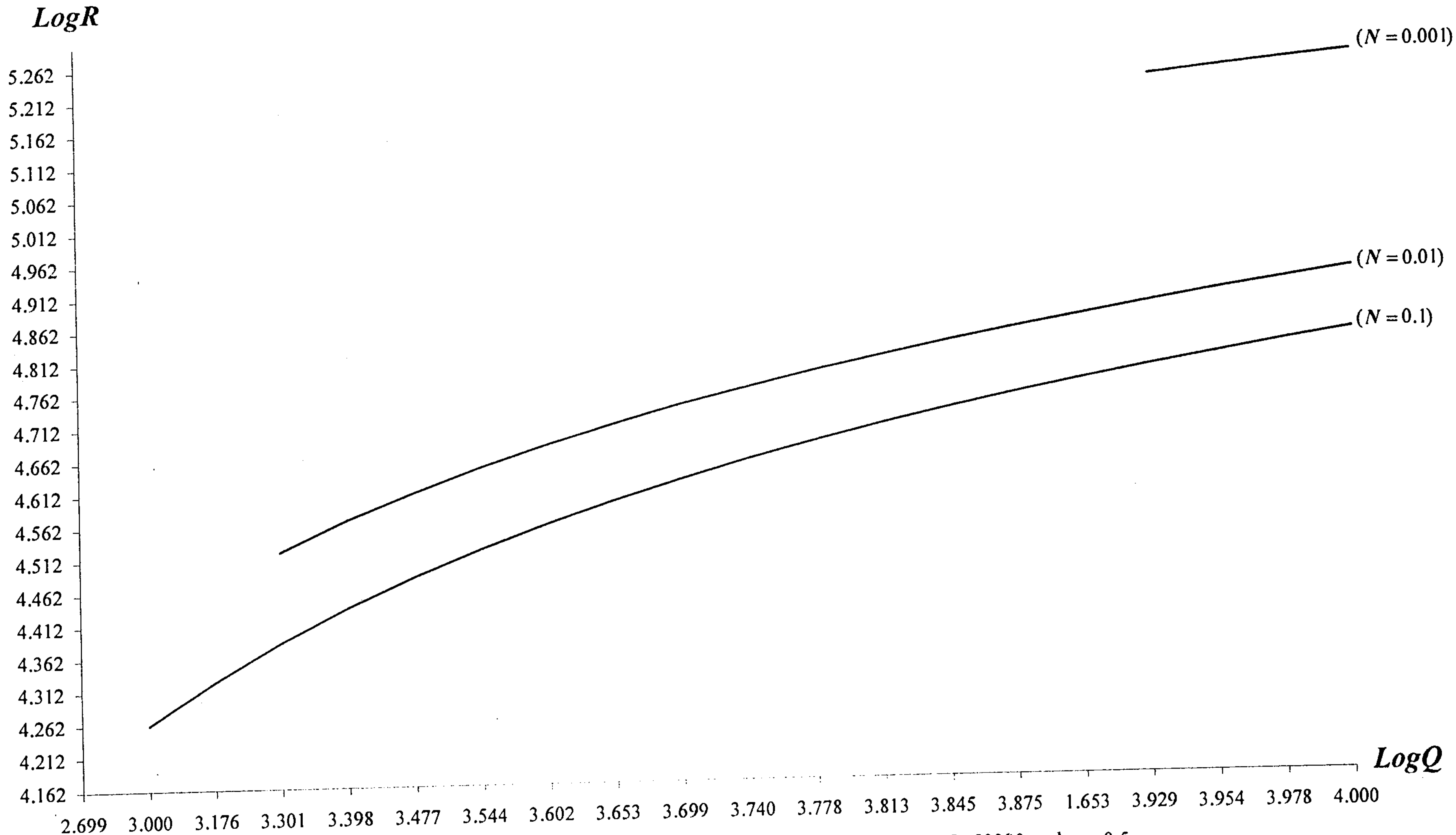

Figure 33. The relation between R and Q for the overstability case when both boundaries are rigid for T=50000 and $\varepsilon = 0.5$.

Table 33. A comparison between the free boundary conditions and the rigid boundary conditions for the overstability case when $T = 10000$ and $\varepsilon = 0.5$.

	(*N*=0.01)				(*N*=0.001)			
Q	a_f	R_f	a_r	R_r	a_f	R_f	a_r	R_r
500	3.338	10571.377	-	-	-	-	-	-
1000	3.542	12776.671	-	-	-	-	-	-
1500	3.743	15012.710	5.184	27978.459	3.235	62445.994	-	-
2000	3.925	17230.072	5.460	32139.890	3.293	65386.094	-	-
2500	4.089	19417.427	5.712	36088.562	3.349	68288.447	-	-
3000	4.238	21573.588	5.921	39874.203	3.402	71156.514	-	-
3500	4.373	23700.270	6.106	43529.319	3.453	73993.269	-	-
4000	4.498	25799.917	6.273	47076.865	3.501	76801.283	-	-
4500	4.613	27874.993	6.421	50533.722	3.548	79582.795	-	-
5000	4.720	29927.766	6.563	53912.652	3.593	82339.766	-	-
5500	4.821	31960.249	6.691	57223.735	3.636	85073.928	-	-
6000	4.916	33974.208	6.812	60474.946	3.678	87786.816	-	-
6500	5.005	35971.189	6.922	63672.816	3.718	90479.799	-	-
7000	5.090	37952.544	7.028	66822.733	3.758	93154.105	-	-
7500	5.170	39919.463	7.129	69929.218	3.796	95810.840	-	-
8000	5.247	41872.991	7.226	72996.127	3.833	98451.003	5.317	164950.220
8500	5.321	43814.058	7.314	76026.751	3.868	101075.502	5.382	169465.660
9000	5.392	45743.488	7.399	79023.909	3.903	103685.165	5.446	173917.555
9500	5.459	47662.019	7.484	81990.129	3.938	106280.746	5.507	178316.274
10000	5.525	49570.314	7.562	84927.572	3.971	108862.939	5.567	182662.773

a_f and R_f are the wave number and the Rayleigh number for the free boundary case.
a_r and R_r are the wave number and the Rayleigh number for the rigid boundary case.

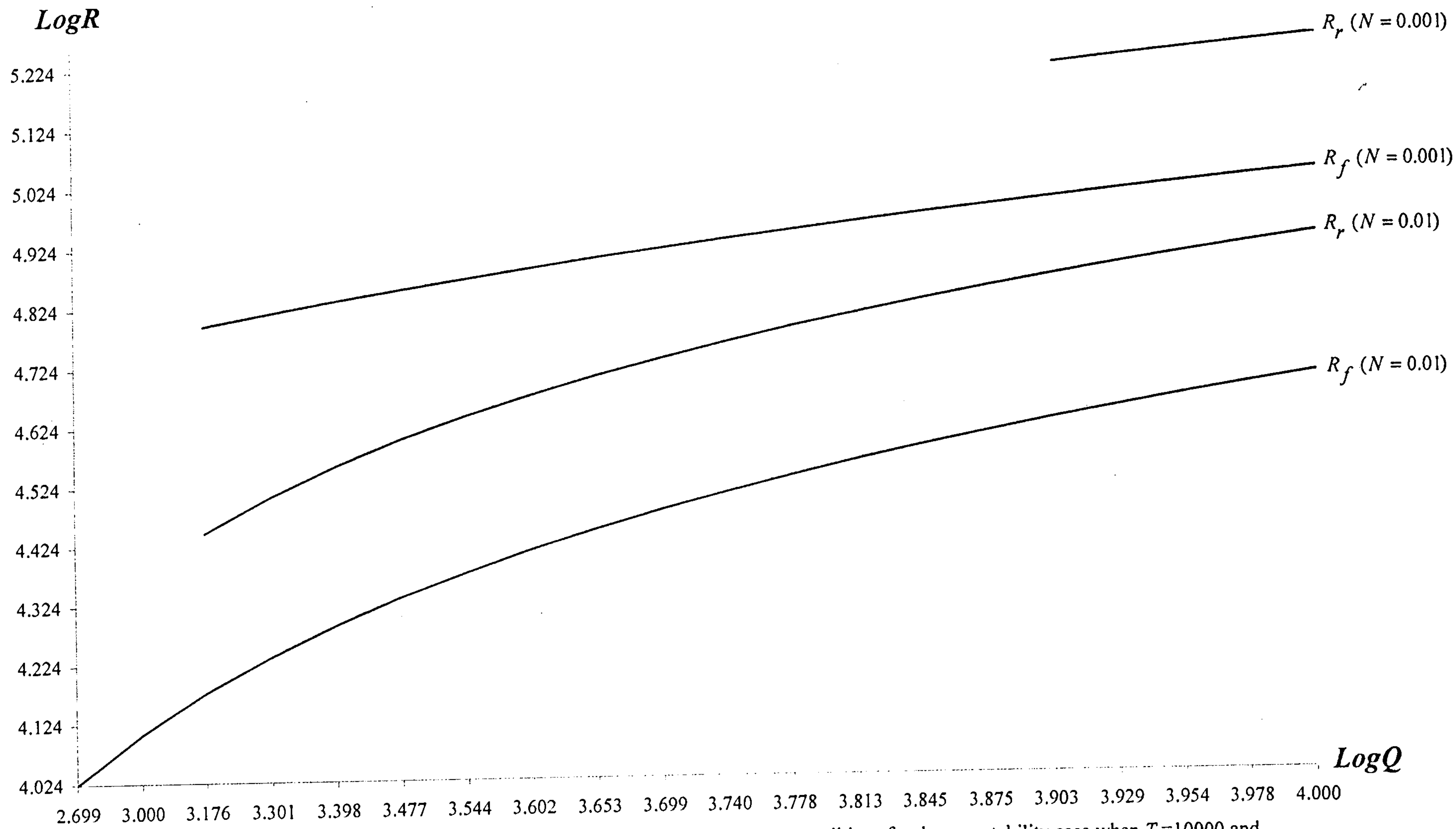

Figure 34. A comparison between the free boundary conditions and the rigid boundary conditions for the overstability case when $T = 10000$ and $\varepsilon = 0.5$.

Appendix (I)

```
      PROGRAM P1

***
***   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***   This Program solves the Benard Problem in the presence
***   of both magnetic field and Coriolis forces using
***   approximation of Chebyshev polynomials.
***   A nag routine F02BJF is used to fined the required
***   eigenvalue. The golden section technique is used to
***   minimize the Rayleigh number over the wave number.
***   The fluid layer is heated from below and the boundaries
***   are free(Linear relationship between the magnetic field
***   and the magnetic induction).
***
***   i.e.        W=D2W=0           on z=0,1
***
***   ______________________________________________________________
***   A, B              : The upper and lower bound of the
***                     : wave number.
***   RALY1, RALY2      : The upper and lower bound of the
***                     : critical Rayleigh number.
***   Q                 : Chandrasekhar number.
***   PR                : Viscous Prandtle number.
***   PM                : Magnetic Prandtle number.
***   Wave              : The wave number.
***   RALY              : The critical Rayleigh number.
***   SEGMAR            : The real part of the eigenvalue.
***   SEGMAI            : The imaginary part of the eigenvalue.
***   ______________________________________________________________
***   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      COMMON RALY1,RALY2, TOL, Q, T, PR, PM
      OPEN (8, FILE='C:\WORK\RESULT' )
***
***   Reading the data
***
      WRITE(*, *)'A, B, RALY1, RALY2, Q, T, PR, PM ='
      READ*, A, B, RALY1, RALY2, Q
      WRITE(8,100) Q
      WRITE(8, 200) PR, PM

***
***   Using the golden section method
***
      TOL= 10.0D0**(-8)
      RG = 0.5D0* (DSQRT(5.0D0)-1.0D0)
      M  = DABS (DLOG(TOL/(B-A))/DLOG(RG))
      X1 = A+ RG * (B-A)
```

```
      CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

      X2 = A + RG **2 * (B-A)

      CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

      DO 10 I=1, M
        IF(R1.GT.R2) THEN
          B = X1
          X1= X2
          R1= R2
          X2 = A+ RG **2 *(B-A)

          CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

        ELSE
          A = X2
          X2= X1
          R2= R1
          X1= A+ RG * (B-A)

          CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

        ENDIF

        R12= DABS(R1-R2)
        IF (R12 .LT. TOL) GOTO 20
10    CONTINUE

20    WAVE = (X1 + X2)/2.0D0
      CALL CHEBY (WAVE, RALY, SEGMAR, SEGMAI)

      WRITE (8, 300) WAVE
      WRITE (8, 400) RALY
      WRITE (8, 500) SEGMAR, SEGMAI

100   FORMAT (//6X, 'Q=',F16.5)
200   FORMAT (//6X, 'PR=', F8.4, 4X, 'PM=', F8.4)
300   FORMAT (//6X, 'A=', F12.5)
400   FORMAT (//6X, 'R=', F19.6 )
500   FORMAT (//6X, 'SEGMAR=', F19.5, ' SEGMAI=', F19.5)

      STOP
      END

***
***   Subroutine to solve the eigenvalue problem
***            (AX=SEGMA BX)
***
***   using the nag routine F02BJF
***
      SUBROUTINE CHEBY (X, RALY, SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (N=20, L1=30, L=6*L1)
```

```
      DIMENSION A(L,L), B(L,L), Z(L,L), XI(L1,L1), D(L1,L1),
     *           D2(L1,L1),V(L1,L1), ALFR(L), ALFI(L),BETA(L),
     *           ITER(L), R(100), SR(L), SI(L),ELANDAR(L),
     *           ELANDAI(L)

      COMMON RALY1, RALY2, TOL, Q, T, PR, PM
      LOGICAL MATZ
      EXTERNAL F02BJF

      A2 = (X/2.0D0)**2
      R(1)=RALY1
      R(2)= RALY2
      N2  = 2*N
      N3  = 3*N
      N4  = 4*N
      N5  = 5*N
      N6  = 6*N
***
***   The identity matrix
***
      DO 2 I=1, N
        DO 1 J=1, N
         IF(J .EQ. I)THEN
           XI(I,J)=1.0D0
         ELSE
           XI(I,J)=0.0D0
         ENDIF
 1      CONTINUE
 2    CONTINUE
***
***   Calling subroutines to construct the derivative matrix (D)
***
***   and the second derivative (D2)
***
      CALL DERIVE (N, D)
      CALL PROD (N,D,D,D2)

      DO 4 I=1, N
        DO 3 J=1, N
          V(I,J)= 4.0D0 * (D2(I,J)-A2 * XI(I,J))
 3      CONTINUE
 4    CONTINUE


      DO 45 K=1, 100
       IF (K.GT.2)THEN
       R(K)=(R(K-2) * SR(K-1) - R(K-1)* SR(K-2))/(SR(K-1)-SR(K-2))
       ENDIF

        RA2= (DSQRT(R(K)) * X * X) * (-1.0D0)

        DO 6 I=1, N6
          DO 5 J=1, N6
            A(I,J)=0.0D0
            B(I,J)=0.0D0
 5        CONTINUE
 6      CONTINUE
```

```
***
***   Building the square matrices A & B
***
***
***   Enter the elements A11 and B11 with the boundary conditions
***

        DO 7 I=1,N
          A(1,I)= 2.0D0 * DBLE(I-1)**2
          A(2,I)=(2.0D0 * DBLE(I-1)**2)  *  (-1.0D0)**(I)
 7      CONTINUE
        DO 9  I=3, N
          DO 8  J=1,N
            A(I,J)=V(I-2,J)
            B(I,J)=XI(I-2,J)
 8          CONTINUE
 9      CONTINUE
***
***   Enter the elements A22 and B22 with the boundary conditions
***
        DO 10 I=1,N
          A(N+1,I+N)= 1.0D0
          A(N+2,I+N)=(-1.0D0)**(I-1)
 10     CONTINUE

        DO 12 I=N+3, N2
          DO 11 J=N+1, N2
            A(I,J)=V(I-N-2,J-N)
            B(I,J)=PM * XI(I-N-2,J-N)
 11         CONTINUE
 12     CONTINUE

***
***   Enter the elements A33 and B33 with the boundary conditions
***
        DO 13 I=1,N
          A(N2+1,I+N2)=1.0D0
          A(N2+2,I+N2)=(-1.0D0)**(I-1)
 13     CONTINUE

        DO 15 I=N2+3, N3
          DO 14 J=N2+1, N3
            A(I,J)=V(I-N2-2,J-N2)
            B(I,J)= XI(I-N2-2,J-N2)
 14         CONTINUE
 15     CONTINUE
***
***   Enter the elements A44 and B44 with the boundary conditions
***
        DO 16 I=1, N
          A(N3+1,N3+I)= 2.0D0 * DBLE(I-1)**2
          A(N3+2,N3+I)=(2.0D0 * DBLE(I-1)**2)  *  (-1.0D0)**(I)
 16     CONTINUE
```

```
      DO 18 I=N3+3, N4
        DO 17 J=N3+1, N4
          A(I,J)=V(I-N3-2,J-N3)
          B(I,J)=PM * XI(I-N3-2,J-N3)
 17     CONTINUE
 18   CONTINUE
***
***  Enter the elements A55 and B55 with the boundary conditions
***
      DO 20 I=N4+3, N5
        DO 19 J=N4+1,N5
          A(N4+1,J)= 1.0D0
          A(N4+2,J)=(-1.0D0)**(J-N4-1)
          A(I,J)   =V(I-N4-2,J-N4)
          B(I,J)   =PR * XI(I-N4-2,J-N4)
 19     CONTINUE
 20   CONTINUE
***
***  Enter the element A66 with the boundary conditions
***
      DO 22 I=N5+3, N6
        DO 21 J=N5+1, N6
          A(N5+1,J)=1.0D0
          A(N5+2,J)=(-1.0D0)**(J-N5-1)
          A(I,J)   =V(I-N5-2,J-N5)
 21     CONTINUE
 22   CONTINUE
***
***  Enter the element B34
***

      DO 24 I=N2+3, N3
        DO 23 J=N3+1, N4
          B(I,J)= (-2.0D0) * PM * D(I-N2-2,J-N3)
 23     CONTINUE
 24   CONTINUE
***
***  Enter the element A12
***

      DO 26 I=3, N
        DO 25 J=N+1, N2
          A(I,J)=2.0D0 * D(I-2,J-N)
 25     CONTINUE
 26   CONTINUE
***
***  Enter the element A16
***

      DO 28 I=3, N
        DO 27 J=N5+1, N6
          A(I,J)=2.0D0 * DSQRT(T) * D(I-2, J-N5)
 27     CONTINUE
 28   CONTINUE
***
***  Enter the element A21
***
```

```
      DO 30 I=N+3, N2
        DO 29 J=1, N
          A(I,J)= 2.0D0 * Q * D(I-N-2,J)
29        CONTINUE
30      CONTINUE
***
***     Enter the element A31
***

      DO 32 I=N2+3, N3
        DO 31 J=1, N
          A(I,J)=(-2.0D0) * DSQRT(T) * D(I-N2-2,J)
31        CONTINUE
32      CONTINUE
***
***     Enter the element A35
***

      DO 34 I=N2+3, N3
        DO 33 J=N4+1, N5
          A(I,J)= RA2 * XI(I-N2-2,J-N4)
33        CONTINUE
34      CONTINUE
***
***     Enter the element A36
***

      DO 36 I=N2+3, N3
        DO 35 J=N5+1, N6
          A(I,J)= (-4.0D0) * Q * D2(I-N2-2,J-N5)
35        CONTINUE
36      CONTINUE
***
***     Enter the element A46
***

      DO 38 I=N3+3, N4
        DO 37 J=N5+1, N6
          A(I,J)= 2.0D0 * Q * D(I-N3-2,J-N5)
37        CONTINUE
38      CONTINUE
***
***     Enter the element A56
***

      DO 40 I=N4+3, N5
        DO 39 J=N5+1, N6
          A(I,J)=DSQRT(R(K)) * XI(I-N4-2,J-N5)
39        CONTINUE
40      CONTINUE
***
***     Enter the element A63
***
```

```
      DO 42 I=N5+3, N6
        DO 41 J=N2+1, N3
         A(I,J)=(-1.0D0) * XI(I-N5-2,J-N2)
41      CONTINUE
42    CONTINUE

      MATZ = .FALSE.
      EPS1 = (10.0D0)**(-15)
      IFAIL= 0
***
***   Calling nag routine to solve the eigenvalue problem
***
      CALL F02BJF(N6,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,IFAIL)

      M=0
      DO 43 I=1, N6
        IF(BETA(I) .NE. 0.0D0) THEN
          M=M+1
          ELANDAR(M)=ALFR(I)/BETA(I)
          ELANDAI(M)=ALFI(I)/BETA(I)
        ENDIF
43    CONTINUE
***
***   Choosing the largest real part of the eigenvalue
***
      ALARGR=ELANDAR(1)

      DO 44 I=2, M
        IF ( ELANDAR(I) .GE. ALARGR) THEN
          ALARGR= ELANDAR(I)
          ALARGI= ELANDAI(I)
        ENDIF
44    CONTINUE

      SR(K)=ALARGR
      SI(K)=ALARGI
      IF (K.GE.2) THEN
        DF=DABS(R(K)-R(K-1))
        IF(DF .LT. TOL) GOTO 46
        ENDIF
45    CONTINUE
46    RALY= R(K)
      SEGMAR=SR(K)
      SEGMAI=SI(K)

      RETURN
      END
```

```
***
***   Subroutine to construct the derivative matrix
***
      SUBROUTINE DERIVE (N, D)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION D(L,L)
      DO 10 I=1,N
        DO 10 J=1,N
          IF((J .GT. I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
            D(I,J)=2.0D0 * DBLE (J-1)
          ELSE
            D(I,J)=0.0D0
          ENDIF
10    CONTINUE


      DO 20 I=1,N
          D(1,I)=0.5D0 * D(1,I)
20    CONTINUE
      RETURN
      END
***
***   Subroutine to multiply two matrices
***
      SUBROUTINE PROD (N,A,B,C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION A(L,L), B(L,L), C(L,L)
      DO 20 I=1,N
        DO 20 J=1,N
          C(I,J)=0.0D0
          DO 10 M=1,N
            C(I,J)=C(I,J)+A(I,M) * B(M,J)
10        CONTINUE
20    CONTINUE
      RETURN
      END
```

Appendix (II)

```
      PROGRAM P2
***
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***   This Program solves the Benard-Porous Problem in the
***   presence of both magnetic field and Coriolis forces using
***   approximation of Chebyshev polynomials.
***   A nag routine F02BJF is used to fined the require eigenvalue.
***   The golden section technique is used to minimize the Rayleigh
***   number over the wave number.
***   The fluid layer is heated from below and the boundaries are
***   free(Linear relationship between the magnetic field and the
***   magnetic induction).
***
***   i.e.      W=D2W=0         on z=0,1
***
***   ______________________________________________________________
***   A, B              : The upper and lower bound of the
***                     : wave number.
***   RALY1, RALY2      : The upper and lower bound of the
***                     : critical Rayleigh number.
***   Q                 : Chandrasekhar number.
***   N1                : (1/N) Inverse of the non-dimensional
***                     : permeability.
***   PR                : Viscous Prandtle number.
***   PM                : Magnetic Prandtle number.
***   Wave              : The wave number.
***   RALY              : The critical Rayleigh number.
***   SEGMAR            : The real part of the eigenvalue.
***   SEGMAI            : The imaginary part of the eigenvalue.
***   ______________________________________________________________
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      REAL N1
      COMMON RALY1,RALY2, TOL, Q, N1,T, PR, PM
      OPEN (8, FILE='C:\WORK\RESULT' )
***
***   Reading the data
***
      WRITE(*, *)'A, B, RALY1, RALY2, Q, N1, T, PR, PM ='
      READ*, A, B, RALY1, RALY2, Q, N1, T, PR, PM

      WRITE(8,100) Q, N1
      WRITE(8, 200) PR, PM
***
***   Using the golden section method
***
      TOL= 10.0D0**(-8)
      RG = 0.5D0* (DSQRT(5.0D0)-1.0D0)
      M  = DABS (DLOG(TOL/(B-A))/DLOG(RG))
      X1 = A+ RG * (B-A)
```

```
      CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

      X2 = A + RG **2 * (B-A)

      CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

      DO 10 I=1, M
        IF(R1.GT.R2) THEN
          B = X1
          X1= X2
          R1= R2
          X2 = A+ RG **2 *(B-A)

          CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

        ELSE
          A = X2
          X2= X1
          R2= R1
          X1= A+ RG * (B-A)

          CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

        ENDIF

        R12= DABS(R1-R2)
        IF (R12 .LT. TOL) GOTO 20
10    CONTINUE

20    WAVE = (X1 + X2)/2.0D0
      CALL CHEBY (WAVE, RALY, SEGMAR, SEGMAI)

      PRINT*, 'THE WAVE NUMBER=', WAVE
      WRITE (8, 300) WAVE
      PRINT*, 'THE CRITICAL RAYLIEGH NUMBER =', RALY
      WRITE (8, 400) RALY
      PRINT*, ' THE REAL PART OF EIGENVALUE=',SEGMAR
      PRINT*, ' THE IMAGINARY PART OF THE EIGENVALUE=', SEGMAI
      WRITE(8, 500) SEGMAR, SEGMAI

100   FORMAT (//6X, 'Q=',F16.5, 4X, 'N=',F18.9)
200   FORMAT (//6X, 'PR=', F8.4, 4X, 'PM=', F8.4)
300   FORMAT (//6X, 'A=', F12.5)
400   FORMAT (//6X, 'R=', F19.6 )
500   FORMAT (//6X, 'SEGMAR=', F19.5, ' SEGMAI=', F19.5)

      STOP
      END
***
***   Subroutine to solve the eigenvalue problem (AX=SEGMA BX)
***   using the nag routine F02BJF
***
      SUBROUTINE CHEBY (X, RALY, SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (N=20, L1=30, L=6*L1)
```

```
      DIMENSION A(L,L), B(L,L), Z(L,L), XI(L1,L1), D(L1,L1),
     *          D2(L1,L1),V(L1,L1), VN(L1,L1), ALFR(L), ALFI(L),
     *          BETA(L),ITER(L), R(100), SR(L), SI(L),ELANDAR(L),
     *          ELANDAI(L)

      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1,T, PR, PM
      LOGICAL MATZ
      EXTERNAL F02BJF

      A2 = (X/2.0D0)**2
      R(1) =RALY1
      R(2)= RALY2
      N2  = 2*N
      N3  = 3*N
      N4  = 4*N
      N5  = 5*N
      N6  = 6*N
***
***   The identity matrix
***
      DO 2 I=1, N
        DO 1 J=1, N
         IF(J .EQ. I)THEN
           XI(I,J)=1.0D0
         ELSE
           XI(I,J)=0.0D0
         ENDIF
 1      CONTINUE
 2    CONTINUE
***
***   Calling subroutines to construct the derivative matrix (D)
***   and the second derivative (D2)
***
      CALL DERIVE (N, D)
      CALL PROD (N,D,D,D2)

      DO 4 I=1, N
        DO 3 J=1, N
          V(I,J)= 4.0D0 * (D2(I,J)-A2 * XI(I,J))
 3      CONTINUE
 4    CONTINUE

      DO 6 I=1, N
        DO 5 J=1, N
         VN(I,J)=V(I,J)-N1 * XI(I,J)
 5      CONTINUE
 6    CONTINUE

      DO 47 K=1, 100
       IF (K.GT.2)THEN
       R(K)=(R(K-2) * SR(K-1) - R(K-1)* SR(K-2))/(SR(K-1)-SR(K-2))
       ENDIF
```

```
      RA2= (DSQRT(R(K)) * X * X) * (-1.0D0)

        DO 8 I=1, N6
          DO 7 J=1, N6
            A(I,J)=0.0D0
            B(I,J)=0.0D0
 7        CONTINUE
 8      CONTINUE
***
***   Building the square matrices A & B
***
***
***   Enter the elements A11 and B11 with the boundary conditions
***

        DO 9 I=1,N
          A(1,I)= 2.0D0 * DBLE((I-1)**2)
          A(2,I)=(2.0D0 * DBLE((I-1)**2))*(-1.0D0)**(I)
 9      CONTINUE
        DO 11 I=3, N
          DO 10 J=1,N
            A(I,J)=VN(I-2,J)
            B(I,J)=XI(I-2,J)
 10         CONTINUE
 11     CONTINUE
***
***   Enter the elements A22 and B22 with the boundary conditions
***
        DO 12 I=1,N
          A(N+1,I+N)= 1.0D0
          A(N+2,I+N)=(-1.0D0)**(I-1)
 12     CONTINUE

        DO 14 I=N+3, N2
          DO 13 J=N+1, N2
            A(I,J)=V(I-N-2,J-N)
            B(I,J)=PM * XI(I-N-2,J-N)
 13         CONTINUE
 14     CONTINUE
***
***   Enter the elements A33 and B33 with the boundary conditions
***
        DO 15 I=1,N
          A(N2+1,I+N2)=1.0D0
          A(N2+2,I+N2)=(-1.0D0)**(I-1)
 15     CONTINUE

        DO 17 I=N2+3, N3
          DO 16 J=N2+1, N3
            A(I,J)=VN(I-N2-2,J-N2)
            B(I,J)= XI(I-N2-2,J-N2)
 16         CONTINUE
 17     CONTINUE
```

```
***
***   Enter the elements A44 and B44 with the boundary conditions
***
      DO 18 I=1, N
        A(N3+1,N3+I)= 2.0D0 * DBLE((I-1)**2)
        A(N3+2,N3+I)=(2.0D0 * DBLE((I-1)**2)) * (-1.0D0)**(I)
 18   CONTINUE
        DO 20 I=N3+3, N4
          DO 19 J=N3+1, N4
            A(I,J)=V(I-N3-2,J-N3)
            B(I,J)=PM * XI(I-N3-2,J-N3)
 19       CONTINUE
 20     CONTINUE
***
***   Enter the elements A55 and B55 with the boundary conditions
***
      DO 22 I=N4+3, N5
        DO 21 J=N4+1,N5
          A(N4+1,J)= 1.0D0
          A(N4+2,J)=(-1.0D0)**(J-N4-1)
          A(I,J)    =V(I-N4-2,J-N4)
          B(I,J)    =PR * XI(I-N4-2,J-N4)
 21     CONTINUE
 22   CONTINUE
***
***   Enter the element A66 with the boundary conditions
***
      DO 24 I=N5+3, N6
        DO 23 J=N5+1, N6
          A(N5+1,J)=1.0D0
          A(N5+2,J)=(-1.0D0)**(J-N5-1)
          A(I,J)    =V(I-N5-2,J-N5)
 23     CONTINUE
 24   CONTINUE
***
***   Enter the element B34
***

      DO 26 I=N2+3, N3
        DO 25 J=N3+1, N4
          B(I,J)= (-2.0D0) * PM * D(I-N2-2,J-N3)
 25     CONTINUE
 26   CONTINUE
***
***   Enter the element A12
***

      DO 28 I=3, N
        DO 27 J=N+1, N2
          A(I,J)=2.0D0 * D(I-2,J-N)
 27     CONTINUE
 28   CONTINUE
```

```
***
***     Enter the element A16
***

        DO 30 I=3, N
          DO 29 J=N5+1, N6
           A(I,J)=2.0D0 * DSQRT(T) * D(I-2, J-N5)
 29       CONTINUE
 30     CONTINUE
***
***     Enter the element A21
***

        DO 32 I=N+3, N2
          DO 31 J=1, N
           A(I,J)= 2.0D0 * Q * D(I-N-2,J)
 31       CONTINUE
 32     CONTINUE
***
***     Enter the element A31
***

        DO 34 I=N2+3, N3
          DO 33 J=1, N
           A(I,J)=(-2.0D0) * DSQRT(T) * D(I-N2-2,J)
 33       CONTINUE
 34     CONTINUE
***
***     Enter the element A35
***

        DO 36 I=N2+3, N3
          DO 35 J=N4+1, N5
           A(I,J)= RA2 * XI(I-N2-2,J-N4)
 35       CONTINUE
 36     CONTINUE
***
***     Enter the element A36
***

        DO 38 I=N2+3, N3
          DO 37 J=N5+1, N6
           A(I,J)= (-4.0D0) * Q * D2(I-N2-2,J-N5)
 37       CONTINUE
 38     CONTINUE
***
***     Enter the element A46
***

        DO 40 I=N3+3, N4
          DO 39 J=N5+1, N6
           A(I,J)= 2.0D0 * Q * D(I-N3-2,J-N5)
 39       CONTINUE
 40     CONTINUE
```

```
***
***     Enter the element A56
***

        DO 42 I=N4+3, N5
          DO 41 J=N5+1, N6
           A(I,J)=DSQRT(R(K)) * XI(I-N4-2,J-N5)
 41       CONTINUE
 42     CONTINUE
***
***     Enter the element A63
***

        DO 44 I=N5+3, N6
          DO 43 J=N2+1, N3
           A(I,J)=(-1.0D0) * XI(I-N5-2,J-N2)
 43       CONTINUE
 44     CONTINUE

        MATZ = .FALSE.
        EPS1 =(10.0D0)**(-15)
        IFAIL= 0
***
***     Calling nag routine to solve the eigenvalue problem
***
        CALL F02BJF(N6,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,IFAIL)

        M=0
        DO 45 I=1, N6
          IF(BETA(I) .NE. 0.0D0) THEN
            M=M+1
            ELANDAR(M)=ALFR(I)/BETA(I)
            ELANDAI(M)=ALFI(I)/BETA(I)
          ENDIF
 45     CONTINUE
***
***     Choosing the largest real part of the eigenvalue
***
        ALARGR=ELANDAR(1)

        DO 46 I=2, M
          IF ( ELANDAR(I) .GE. ALARGR) THEN
            ALARGR= ELANDAR(I)
            ALARGI= ELANDAI(I)
          ENDIF
 46     CONTINUE

        SR(K)=ALARGR
        SI(K)=ALARGI
        IF (K.GE.2) THEN
          DF=DABS(R(K)-R(K-1))
          IF(DF .LT. TOL) GOTO 48
          ENDIF
 47     CONTINUE
 48     RALY= R(K)
```

```
      SEGMAR=SR(K)
      SEGMAI=SI(K)
      RETURN
      END
***
***   Subroutine to construct the derivative matrix
***
      SUBROUTINE DERIVE (N, D)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION D(L,L)
      DO 10 I=1,N
        DO 10 J=1,N
          IF((J .GT. I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
            D(I,J)=2.0D0 * DBLE (J-1)
          ELSE
            D(I,J)=0.0D0
          ENDIF
 10   CONTINUE
      DO 20 I=1,N
        D(1,I)=0.5D0 * D(1,I)
 20   CONTINUE
      RETURN
      END
***
***   Subroutine to multiply two matrices
***
      SUBROUTINE PROD (N,A,B,C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION A(L,L), B(L,L), C(L,L)
      DO 20 I=1,N
        DO 20 J=1,N
          C(I,J)=0.0D0
          DO 10 M=1,N
            C(I,J)=C(I,J)+A(I,M) * B(M,J)
 10       CONTINUE
 20   CONTINUE
      RETURN
      END
```

Appendix (III)

```
      PROGRAM P3
***
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***   This Program solves the Benard-Porous Problem in the
***   presence of both magnetic field and Coriolis forces using
***   approximation of Chebyshev polynomials.
***   A nag routine F02BJF is used to fined the required eigenvalue.
***   The golden section technique is used to minimize the Rayleigh
***   number over the wave number.
***   The fluid layer is heated from below and the boundaries are
***   rigid (Linear relationship between the magnetic field and the
***   magnetic induction).
***
***   i.e.        W=DW=0          on z=0,1
***
***   ____________________________________________________________
***   A, B               : The upper and lower bound of the
***                      : wave number.
***   RALY1, RALY2       : The upper and lower bound of the
***                      critical Rayleigh number.
***   Q                  : Chandrasekhar number.
***   N1                 : (1/N) Inverse of the non-dimensional
***                      : permeability.
***   PR                 : Viscous Prandtle number.
***   PM                 : Magnetic Prandtle number.
***   Wave               : The wave number.
***   RALY               : The critical Rayleigh number.
***   SEGMAR             : The real part of the eigenvalue.
***   SEGMAI             : The imaginary part of the eigenvalue.
***   ____________________________________________________________
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      REAL N1
      COMMON RALY1,RALY2, TOL, Q, N1,T, PR, PM
      OPEN (8, FILE='C:\WORK\RESULT' )
***
***   Reading the data
***
      WRITE(*, *)'A, B, RALY1, RALY2, Q, N1, T, PR, PM ='
      READ*, A, B, RALY1, RALY2, Q, N1, T, PR, PM

      WRITE(8,100) Q, N1
      WRITE(8, 200) PR, PM
***
***   Using the golden section method
***
      TOL= 10.0D0**(-8)
      RG = 0.5D0* (DSQRT(5.0D0)-1.0D0)
      M  = DABS (DLOG(TOL/(B-A))/DLOG(RG))
      X1 = A+ RG * (B-A)
```

```
      CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

      X2 = A + RG **2 * (B-A)

      CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

      DO 10 I=1, M
        IF(R1.GT.R2) THEN
          B = X1
          X1= X2
          R1= R2
          X2 = A+ RG **2 *(B-A)

          CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

        ELSE
          A = X2
          X2= X1
          R2= R1
          X1= A+ RG * (B-A)

          CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

        ENDIF

        R12= DABS(R1-R2)
        IF (R12 .LT. TOL) GOTO 20
10    CONTINUE

20    WAVE = (X1 + X2)/2.0D0
      CALL CHEBY (WAVE, RALY, SEGMAR, SEGMAI)

      PRINT*, 'THE WAVE NUMBER=', WAVE
      WRITE (8, 300) WAVE
      PRINT*, 'THE CRITICAL RAYLIEGH NUMBER =', RALY
      WRITE (8, 400) RALY
      PRINT*, ' THE REAL PART OF EIGENVALUE=',SEGMAR
      PRINT*, ' THE IMAGINARY PART OF THE EIGENVALUE=', SEGMAI
      WRITE(8, 500) SEGMAR, SEGMAI

100   FORMAT (//6X, 'Q=',F16.5, 4X, 'N=',F18.9)
200   FORMAT (//6X, 'PR=', F8.4, 4X, 'PM=', F8.4)
300   FORMAT (//6X, 'A=', F12.5)
400   FORMAT (//6X, 'R=', F19.6 )
500   FORMAT (//6X, 'SEGMAR=', F19.5, ' SEGMAI=', F19.5)

      STOP
      END
***
***   Subroutine to solve the eigenvalue problem (AX=SEGMA BX)
***   using the nag routine F02BJF
***
      SUBROUTINE CHEBY (X, RALY, SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (N=20, L1=30, L=6*L1)
```

```
      DIMENSION A(L,L), B(L,L), Z(L,L), XI(L1,L1), D(L1,L1),
     *          D2(L1,L1),V(L1,L1), VN(L1,L1), ALFR(L), ALFI(L),
     *          BETA(L),ITER(L), R(100), SR(L), SI(L),ELANDAR(L),
     *          ELANDAI(L)

      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1,T, PR, PM
      LOGICAL MATZ
      EXTERNAL F02BJF

      A2 = (X/2.0D0)**2
      R(1) =RALY1
      R(2)= RALY2
      N2  = 2*N
      N3  = 3*N
      N4  = 4*N
      N5  = 5*N
      N6  = 6*N
***
***   The identity matrix
***
      DO 2 I=1, N
        DO 1 J=1, N
         IF(J .EQ. I)THEN
           XI(I,J)=1.0D0
         ELSE
           XI(I,J)=0.0D0
         ENDIF
 1      CONTINUE
 2    CONTINUE
***
***   Calling subroutines to construct the derivative matrix (D)
***   and the second derivative (D2)
***
      CALL DERIVE (N, D)
      CALL PROD (N,D,D,D2)

      DO 4 I=1, N
        DO 3 J=1, N
          V(I,J)= 4.0D0 * (D2(I,J)-A2 * XI(I,J))
 3      CONTINUE
 4    CONTINUE

      DO 6 I=1, N
        DO 5 J=1, N
         VN(I,J)=V(I,J)-N1 * XI(I,J)
 5      CONTINUE
 6    CONTINUE

      DO 47 K=1, 100
       IF (K.GT.2)THEN
       R(K)=(R(K-2) * SR(K-1) - R(K-1)* SR(K-2))/(SR(K-1)-SR(K-2))
       ENDIF
```

```
      RA2= (DSQRT(R(K)) * X * X) * (-1.0D0)

        DO 8 I=1, N6
          DO 7 J=1, N6
            A(I,J)=0.0D0
            B(I,J)=0.0D0
7         CONTINUE
8       CONTINUE
***
***   Building the square matrices A & B
***
***
***   Enter the elements A11 and B11 with the boundary conditions
***
        DO 9 I=1,N
          A(1,I)= 1.0D0
          A(2,I)=(-1.0D0)**(I-1)
9       CONTINUE
        DO 11 I=3, N
          DO 10 J=1,N
            A(I,J)=VN(I-2,J)
            B(I,J)=XI(I-2,J)
10          CONTINUE
11      CONTINUE
***
***   Enter the elements A22 and B22 with the boundary conditions
***
        DO 12 I=1,N
          A(N+1,I+N)= 2.0D0 * DBLE((I-1)**2
          A(N+2,I+N)=(2.0D0 * DBLE((I-1)**2))*(-1.0D0)**(I)
12      CONTINUE

        DO 14 I=N+3, N2
          DO 13 J=N+1, N2
            A(I,J)=V(I-N-2,J-N)
            B(I,J)=PM * XI(I-N-2,J-N)
13          CONTINUE
14      CONTINUE
***
***   Enter the elements A33 and B33
***
        DO 15 I=1,N
          A(N2+1,I+N2)=0.0D0
          A(N2+2,I+N2)=0.0D0
15      CONTINUE

        DO 17 I=N2+3, N3
          DO 16 J=N2+1, N3
            A(I,J)=VN(I-N2-2,J-N2)
            B(I,J)= XI(I-N2-2,J-N2)
16          CONTINUE
17      CONTINUE
```

```
***
***   Enter the elements A44 and B44 with the boundary conditions
***
        DO 18 I=1, N
          A(N3+1,N3+I)= 1.0D0
          A(N3+2,N3+I)=(-1.0D0)**(I-1)
 18     CONTINUE
          DO 20 I=N3+3, N4
            DO 19 J=N3+1, N4
              A(I,J)=V(I-N3-2,J-N3)
              B(I,J)=PM * XI(I-N3-2,J-N3)
 19         CONTINUE
 20       CONTINUE
***
***   Enter the elements A55 and B55 with the boundary conditions
***
        DO 22 I=N4+3, N5
          DO 21 J=N4+1,N5
            A(N4+1,J)= 1.0D0
            A(N4+2,J)=(-1.0D0)**(J-N4-1)
            A(I,J)    =V(I-N4-2,J-N4)
            B(I,J)    =PR * XI(I-N4-2,J-N4)
 21       CONTINUE
 22     CONTINUE
***
***   Enter the element A66 with the boundary conditions
***
        DO 24 I=N5+3, N6
          DO 23 J=N5+1, N6
            A(N5+1,J)=1.0D0
            A(N5+2,J)=(-1.0D0)**(J-N5-1)
            A(I,J)    =V(I-N5-2,J-N5)
 23       CONTINUE
 24     CONTINUE
***
***   Enter the element B34
***

        DO 26 I=N2+3, N3
          DO 25 J=N3+1, N4
            B(I,J)= (-2.0D0) * PM * D(I-N2-2,J-N3)
 25       CONTINUE
 26     CONTINUE
***
***   Enter the element A12
***

        DO 28 I=3, N
          DO 27 J=N+1, N2
            A(I,J)=2.0D0 * D(I-2,J-N)
 27       CONTINUE
 28     CONTINUE
```

```
***
***    Enter the element A16
***

       DO 30 I=3, N
         DO 29 J=N5+1, N6
          A(I,J)=2.0D0 * DSQRT(T) * D(I-2, J-N5)
 29      CONTINUE
 30    CONTINUE
***
***    Enter the element A21
***

       DO 32 I=N+3, N2
         DO 31 J=1, N
          A(I,J)= 2.0D0 * Q * D(I-N-2,J)
 31      CONTINUE
 32    CONTINUE
***
***    Enter the element A31
***

       DO 34 I=N2+3, N3
         DO 33 J=1, N
          A(I,J)=(-2.0D0) * DSQRT(T) * D(I-N2-2,J)
 33      CONTINUE
 34    CONTINUE
***
***    Enter the element A35
***

       DO 36 I=N2+3, N3
         DO 35 J=N4+1, N5
          A(I,J)= RA2 * XI(I-N2-2,J-N4)
 35      CONTINUE
 36    CONTINUE
***
***    Enter the element A36 with the boundary conditions
***

       DO 38 I=N2+3, N3
         DO 37 J=N5+1, N6
         A(N2+1,J)= 2.0D0 * DBLE((I-1)**2)
         A(N2+2,J)=(2.0D0 * DBLE((I-1)**2))*(-1.0D0)**(I)
         A(I,J)= (-4.0D0) * Q * D2(I-N2-2,J-N5)
 37      CONTINUE
 38    CONTINUE
***
***    Enter the element A46
***

       DO 40 I=N3+3, N4
         DO 39 J=N5+1, N6
          A(I,J)= 2.0D0 * Q * D(I-N3-2,J-N5)
 39      CONTINUE
 40    CONTINUE
```

```
***
***    Enter the element A56
***

        DO 42 I=N4+3, N5
          DO 41 J=N5+1, N6
           A(I,J)=DSQRT(R(K)) * XI(I-N4-2,J-N5)
 41       CONTINUE
 42     CONTINUE
***
***    Enter the element A63
***

        DO 44 I=N5+3, N6
          DO 43 J=N2+1, N3
           A(I,J)=(-1.0D0) * XI(I-N5-2,J-N2)
 43       CONTINUE
 44     CONTINUE

        MATZ = .FALSE.
        EPS1 =(10.0D0)**(-15)
        IFAIL= 0
***
***    Calling nag routine to solve the eigenvalue problem
***
       CALL F02BJF(N6,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,IFAIL)

        M=0
        DO 45 I=1, N6
          IF(BETA(I) .NE. 0.0D0) THEN
            M=M+1
            ELANDAR(M)=ALFR(I)/BETA(I)
            ELANDAI(M)=ALFI(I)/BETA(I)
          ENDIF
 45     CONTINUE
***
***     Choosing the largest real part of the eigenvalue
***
        ALARGR=ELANDAR(1)

        DO 46 I=2, M
          IF ( ELANDAR(I) .GE. ALARGR) THEN
            ALARGR= ELANDAR(I)
            ALARGI= ELANDAI(I)
          ENDIF
 46     CONTINUE

        SR(K)=ALARGR
        SI(K)=ALARGI
        IF (K.GE.2) THEN
          DF=DABS(R(K)-R(K-1))
          IF(DF .LT. TOL) GOTO 48
          ENDIF
 47   CONTINUE
 48   RALY= R(K)
```

```
      SEGMAR=SR(K)
      SEGMAI=SI(K)

      RETURN
      END
***
***   Subroutine to construct the derivative matrix
***
      SUBROUTINE DERIVE (N, D)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION D(L,L)
      DO 10 I=1,N
        DO 10 J=1,N
          IF((J .GT. I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
            D(I,J)=2.0D0 * DBLE (J-1)
          ELSE
            D(I,J)=0.0D0
          ENDIF
 10   CONTINUE
      DO 20 I=1,N
        D(1,I)=0.5D0 * D(1,I)
 20   CONTINUE
      RETURN
      END
***
***   Subroutine to multiply two matrices
***
      SUBROUTINE PROD (N,A,B,C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION A(L,L), B(L,L), C(L,L)
      DO 20 I=1,N
        DO 20 J=1,N
          C(I,J)=0.0D0
          DO 10 M=1,N
            C(I,J)=C(I,J)+A(I,M) * B(M,J)
 10       CONTINUE
 20   CONTINUE
      RETURN
      END
```

Appendix (IV)

```
      PROGRAM P4
***
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***   This Program solves the Benard-Porous Problem in the
***   presence of both magnetic field and Coriolis forces using
***   approximation of Chebyshev polynomials.
***   A nag routine F02BJF is used to fined the required eigenvalue.
***   The golden section technique is used to minimize the Rayleigh
***   number over the wave number.
***   The fluid layer is heated from below and the boundaries are
***   free(Non-linear relationship between the magnetic field and the
***   magnetic induction)
***
***   i.e.      W=D2W=0          on z=0,1
***
***   ______________________________________________________________
***   A, B              : The upper and lower bound of the
***                     : wave number.
***   RALY1, RALY2      : The upper and lower bound of the
***                     : critical Rayleigh number.
***   Q                 : Chandrasekhar number.
***   N1                : (1/N) Inverse of the non-dimensional
***                     : permeability.
***   PR                : Viscous Prandtle number.
***   PM                : Magnetic Prandtle number.
***   EPSLON            : Non-dimensional parameter which measure the
***                     : strength of non-linearity.
***   Wave              : The wave number.
***   RALY              : The critical Rayleigh number.
***   SEGMAR            : The real part of the eigenvalue.
***   SEGMAI            : The imaginary part of the eigenvalue.
***   ______________________________________________________________
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1, T, PR, PM, EPSLON
      OPEN (8, FILE='C:\WORK\RESULT')
***
***   Reading the data
***
      WRITE(*, *)'A, B, RALY1, RALY2, Q, N1, T, PR, PM, EPSLON ='
      READ*, A, B, RALY1, RALY2, Q, N1, T, PR, PM, EPSLON
      WRITE(8,100) T, EPSLON
      WRITE(8,200) Q, N1
      WRITE(8,300) PR, PM
***
***   Using the golden section method
***
      TOL= 10.0D0**(-8)
      RG = 0.5D0* (DSQRT(5.0D0)-1.0D0)
      M  = DABS (DLOG(TOL/(B-A))/DLOG(RG))
```

```
      X1 = A+ RG * (B-A)

      CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

      X2 = A + RG **2 * (B-A)

      CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

      DO 10 I=1, M
        IF(R1.GT.R2) THEN
          B = X1
          X1= X2
          R1= R2
          X2 = A+ RG **2 *(B-A)

          CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

        ELSE
          A = X2
          X2= X1
          R2= R1
          X1= A+ RG * (B-A)

          CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

        ENDIF

        R12= DABS(R1-R2)
        IF (R12 .LT. TOL) GOTO 20
10    CONTINUE

20    WAVE = (X1 + X2)/2.0D0
      CALL CHEBY (WAVE, RALY, SEGMAR, SEGMAI)

      WRITE (8,400) WAVE
      WRITE (8,500)'THE CRITICAL RAYLIEGH NUMBER =', RALY
      WRITE (8,600) SEGMAR
      WRITE (8,700) SEGMAI
100   FORMAT (//6X, 'T=', F16.5, 4X, 'EPSLON=', F8.4)
200   FORMAT (//6X, 'Q=', F18.7, 4X, 'N=', F18.9)
300   FORMAT (//6X, 'PR=', F8.4, 4X, 'PM=', F8.4)
400   FORMAT (//6X, 'A=', F12.5)
500   FORMAT (//6X, 'R=', F19.6 )
600   FORMAT (//6X, 'SEGMAR=', F19.5)
700   FORMAT (//6X, 'SEGMAI=', F19.5)

      STOP
      END
***
***   Subroutine to solve the eigenvalue problem (AX=SEGMA BX)
***   using the nag routine F02BJF
***
      SUBROUTINE CHEBY (X, RALY, SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (N=20, L1=30, L=6*L1)
```

```
      DIMENSION A(L,L), B(L,L), Z(L,L), XI(L1,L1), D(L1,L1),
     *           D2(L1,L1),V(L1,L1), VN(L1,L1), ALFR(L), ALFI(L),
     *           BETA(L),ITER(L), R(100), SR(L), SI(L),ELANDAR(L),
     *           ELANDAI(L)

      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1, T, PR, PM, EPSLON
      LOGICAL MATZ
      EXTERNAL F02BJF

      A2 = (X/2.0D0)**2
      EA2= (X**2) * EPSLON
      R(1) =RALY1
      R(2)= RALY2
      N2  = 2*N
      N3  = 3*N
      N4  = 4*N
      N5  = 5*N
      N6  = 6*N
***
***   The identity matrix
***
      DO 2 I=1, N
        DO 1 J=1, N
          IF(J .EQ. I)THEN
            XI(I,J)=1.0D0
          ELSE
            XI(I,J)=0.0D0
          ENDIF
 1      CONTINUE
 2    CONTINUE
***
***   Calling subroutines to find the derivative matrix (D) and
***   the second derivative (D2)
***
      CALL DERIVE (N, D)
      CALL PROD (N,D,D,D2)
***
***   Building the matrices A and B and applying the boundary
***   conditions
***
      DO 4 I=1, N
        DO 3 J=1, N
          V(I,J)= 4.0D0 * (D2(I,J)-A2 * XI(I,J))
 3      CONTINUE
 4    CONTINUE

      DO 6 I=1, N
        DO 5 J=1, N
         VN(I,J)=V(I,J)-N1 * XI(I,J)
 5      CONTINUE
 6    CONTINUE
```

```
      DO 47 K=1, 100
       IF (K.GT.2)THEN
       R(K)=(R(K-2) * SR(K-1) - R(K-1)* SR(K-2))/(SR(K-1)-SR(K-2))
       ENDIF

        RA2= (DSQRT(R(K)) * X * X) * (-1.0D0)

        DO 8 I=1, N6
          DO 7 J=1, N6
            A(I,J)=0.0D0
            B(I,J)=0.0D0
 7        CONTINUE
 8      CONTINUE
***
***   Writing the elements A11 and B11
***
        DO 9 I=1,N
          A(1,I)= 2.0D0 * DBLE((I-1)**2)
          A(2,I)=(2.0D0 * DBLE((I-1)**2))*(-1.0D0)**(I)
 9      CONTINUE
        DO 11 I=3, N
          DO 10 J=1,N
            A(I,J)=VN(I-2,J)
            B(I,J)=XI(I-2,J)
 10         CONTINUE
 11     CONTINUE
***
***   Writing the elements A22 and B22
***

        DO 12 I=1,N
          A(N+1,I+N)= 1.0D0
          A(N+2,I+N)=(-1.0D0)**(I-1)
 12     CONTINUE

        DO 14 I=N+3, N2
          DO 13 J=N+1, N2
            A(I,J)=V(I-N-2,J-N)
            B(I,J)=PM * XI(I-N-2,J-N)
 13       CONTINUE
 14     CONTINUE
***
***   Writing the elements A33 and B33
***

        DO 15 I=1,N
          A(N2+1,I+N2)=1.0D0
          A(N2+2,I+N2)=(-1.0D0)**(I-1)
 15     CONTINUE

        DO 17 I=N2+3, N3
          DO 16 J=N2+1, N3
            A(I,J)=VN(I-N2-2,J-N2)
            B(I,J)= XI(I-N2-2,J-N2)
 16       CONTINUE
 17     CONTINUE
```

```
***
***   Writing the elements A44 and B44
***

        DO 18 I=1, N
          A(N3+1,N3+I)= 2.0D0 * DBLE((I-1)**2)
          A(N3+2,N3+I)=(2.0D0 * DBLE((I-1)**2)) * (-1.0D0)**(I)
 18     CONTINUE
          DO 20 I=N3+3, N4
            DO 19 J=N3+1, N4
              A(I,J)=V(I-N3-2,J-N3)-(EA2 * XI(I-N3-2, J-N3))
              B(I,J)=PM * XI(I-N3-2,J-N3)
 19         CONTINUE
 20       CONTINUE
***
***   Writing the elements A55 and B55
***

        DO 22 I=N4+3, N5
          DO 21 J=N4+1,N5
            A(N4+1,J)= 1.0D0
            A(N4+2,J)=(-1.0D0)**(J-N4-1)
            A(I,J)    =V(I-N4-2,J-N4)
            B(I,J)    =PR * XI(I-N4-2,J-N4)
 21       CONTINUE
 22     CONTINUE
***
***   Writing the element A66
***
        DO 24 I=N5+3, N6
          DO 23 J=N5+1, N6
            A(N5+1,J)=1.0D0
            A(N5+2,J)=(-1.0D0)**(J-N5-1)
            A(I,J)    =V(I-N5-2,J-N5)
 23       CONTINUE
 24     CONTINUE
***
***   Writing the element  B34
***
        DO 26 I=N2+3, N3
          DO 25 J=N3+1, N4
            B(I,J)= (-2.0D0) * PM * D(I-N2-2,J-N3)
 25       CONTINUE
 26     CONTINUE
***
***   Writing the element   A12
***
        DO 28 I=3, N
          DO 27 J=N+1, N2
            A(I,J)=2.0D0 * D(I-2,J-N)
 27       CONTINUE
 28     CONTINUE
***
***   Writing the element  A16
***
```

```
      DO 30 I=3, N
        DO 29 J=N5+1, N6
         A(I,J)=2.0D0 * DSQRT(T) * D(I-2, J-N5)
 29     CONTINUE
 30   CONTINUE
***
***  Writing the element  A21
***
      DO 32 I=N+3, N2
        DO 31 J=1, N
         A(I,J)= 2.0D0 * Q * D(I-N-2,J)
 31     CONTINUE
 32   CONTINUE
***
***  Writing the element  A31
***
      DO 34 I=N2+3, N3
        DO 33 J=1, N
         A(I,J)=(-2.0D0) * DSQRT(T) * D(I-N2-2,J)
 33     CONTINUE
 34   CONTINUE
***
***  Writing the element  A35
***
      DO 36 I=N2+3, N3
        DO 35 J=N4+1, N5
         A(I,J)= RA2 * XI(I-N2-2,J-N4)
 35     CONTINUE
 36   CONTINUE
***
***  Writing the element  A36
***
      DO 38 I=N2+3, N3
        DO 37 J=N5+1, N6
         A(I,J)= (-4.0D0) * Q * D2(I-N2-2,J-N5)
 37     CONTINUE
 38   CONTINUE
***
***  Writing the element  A46
***
      DO 40 I=N3+3, N4
        DO 39 J=N5+1, N6
         A(I,J)= 2.0D0 * Q * D(I-N3-2,J-N5)
 39     CONTINUE
 40   CONTINUE
***
***  Writing the element  A56
***
      DO 42 I=N4+3, N5
        DO 41 J=N5+1, N6
         A(I,J)=DSQRT(R(K)) * XI(I-N4-2,J-N5)
 41     CONTINUE
 42   CONTINUE
```

```
***
***   Writing the element   A63
***
        DO 44 I=N5+3, N6
          DO 43 J=N2+1, N3
           A(I,J)=(-1.0D0) * XI(I-N5-2,J-N2)
 43       CONTINUE
 44     CONTINUE

        MATZ = .FALSE.
        EPS1 = (10.0D0)**(-15)
        IFAIL= 0
***
***   Calling nag routine to solve the eigenvalue problem
***
      CALL F02BJF(N6,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,IFAIL)

        M=0
        DO 45 I=1, N6
          IF(BETA(I) .NE. 0.0D0) THEN
            M=M+1
            ELANDAR(M)=ALFR(I)/BETA(I)
            ELANDAI(M)=ALFI(I)/BETA(I)
          ENDIF
 45     CONTINUE
***
***     Choosing the largest real part of the eigenvalue
***
        ALARGR=ELANDAR(1)

        DO 46 I=2, M
          IF ( ELANDAR(I) .GE. ALARGR) THEN
            ALARGR= ELANDAR(I)
            ALARGI= ELANDAI(I)
          ENDIF
 46     CONTINUE

        SR(K)=ALARGR
        SI(K)=ALARGI
        IF (K.GE.2) THEN
          DF=DABS(R(K)-R(K-1))
          IF(DF .LT. TOL) GOTO 48
          ENDIF
 47   CONTINUE
 48   RALY= R(K)
      SEGMAR=SR(K)
      SEGMAI=SI(K)

      RETURN
      END
```

```
***
***   Subroutine to construct the derivative matrix
***
      SUBROUTINE DERIVE (N, D)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION D(L,L)
      DO 10 I=1,N
        DO 10 J=1,N
          IF((J .GT. I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
            D(I,J)=2.0D0 * DBLE (J-1)
          ELSE
            D(I,J)=0.0D0
          ENDIF
 10   CONTINUE
      DO 20 I=1,N
        D(1,I)=0.5D0 * D(1,I)
 20   CONTINUE
      RETURN
      END
***
***   Subroutine to multiply two matrices
***
      SUBROUTINE PROD (N,A,B,C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION A(L,L), B(L,L), C(L,L)
      DO 20 I=1,N
        DO 20 J=1,N
          C(I,J)=0.0D0
          DO 10 M=1,N
            C(I,J)=C(I,J)+A(I,M) * B(M,J)
 10       CONTINUE
 20   CONTINUE
      RETURN
      END
```

Appendix (V)

```
      PROGRAM P5
***
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***   This Program solves the Benard-Porous Problem in the
***   presence of both magnetic field and Coriolis forces using
***   approximation of Chebyshev polynomials.
***   A nag routine F02BJF is used to fined the required eigenvalue.
***   The golden section technique is used to minimize the Rayleigh
***   number over the wave number.
***   The fluid layer is heated from below and the boundaries are
***   rigid (Non-linear relationship between the magnetic field and
***    the magnetic induction)
***
***   i.e.       W=DW=0          on z=0,1
***
***   ____________________________________________________________
***   A, B              : The upper and lower bound of the
***                     : wave number.
***   RALY1, RALY2      : The upper and lower bound of the
***                     : critical Rayleigh number.
***   Q                 : Chandrasekhar number.
***   N1                : (1/N) Inverse of the non-dimensional
***                     : permeability.
***   PR                : Viscous Prandtle number.
***   PM                : Magnetic Prandtle number.
***   EPSLON            : Non-dimensional parameter which measure the
***                     : strength of non-linearity.
***   Wave              : The wave number.
***   RALY              : The critical Rayleigh number.
***   SEGMAR            : The real part of the eigenvalue.
***   SEGMAI            : The imaginary part of the eigenvalue.
***   ____________________________________________________________
***   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
***
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1, T, PR, PM, EPSLON
      OPEN (8, FILE='C:\WORK\RESULT')
***
***   Reading the data
***
      WRITE(*, *)'A, B, RALY1, RALY2, Q, N1, T, PR, PM, EPSLON ='
      READ*, A, B, RALY1, RALY2, Q, N1, T, PR, PM, EPSLON
      WRITE(8,100) T, EPSLON
      WRITE(8,200) Q, N1
      WRITE(8,300) PR, PM
***
***   Using the golden section method
***
      TOL= 10.0D0**(-8)
      RG = 0.5D0* (DSQRT(5.0D0)-1.0D0)
```

```
      M  = DABS (DLOG(TOL/(B-A))/DLOG(RG))

      X1 = A+ RG * (B-A)

      CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

      X2 = A + RG **2 * (B-A)

      CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

      DO 10 I=1, M
        IF(R1.GT.R2) THEN
          B = X1
          X1= X2
          R1= R2
          X2 = A+ RG **2 *(B-A)

          CALL CHEBY (X2, R2, SEGMAR2, SEGMAI2)

        ELSE
          A = X2
          X2= X1
          R2= R1
          X1= A+ RG * (B-A)

          CALL CHEBY (X1, R1, SEGMAR1, SEGMAI1)

        ENDIF

        R12= DABS(R1-R2)
        IF (R12 .LT. TOL) GOTO 20
10    CONTINUE

20    WAVE = (X1 + X2)/2.0D0
      CALL CHEBY (WAVE, RALY, SEGMAR, SEGMAI)

      WRITE (8,400) WAVE
      WRITE (8,500)'THE CRITICAL RAYLIEGH NUMBER =', RALY
      WRITE (8,600) SEGMAR
      WRITE (8,700) SEGMAI

100   FORMAT (//6X, 'T=', F16.5, 4X, 'EPSLON=', F8.4)
200   FORMAT (//6X, 'Q=', F18.7, 4X, 'N=', F18.9)
300   FORMAT (//6X, 'PR=', F8.4, 4X, 'PM=', F8.4)
400   FORMAT (//6X, 'A=', F12.5)
500   FORMAT (//6X, 'R=', F19.6 )
600   FORMAT (//6X, 'SEGMAR=', F19.5)
700   FORMAT (//6X, 'SEGMAI=', F19.5)

      STOP
      END
***
***   Subroutine to solve the eigenvalue problem (AX=SEGMA BX)
***   using the nag routine F02BJF
***
      SUBROUTINE CHEBY (X, RALY, SEGMAR,SEGMAI)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
```

```
      PARAMETER (N=20, L1=30, L=6*L1)
      DIMENSION A(L,L), B(L,L), Z(L,L), XI(L1,L1), D(L1,L1),
     *          D2(L1,L1),V(L1,L1), VN(L1,L1), ALFR(L), ALFI(L),
     *          BETA(L),ITER(L), R(100), SR(L), SI(L),ELANDAR(L),
     *          ELANDAI(L)

      REAL N1
      COMMON RALY1, RALY2, TOL, Q, N1, T, PR, PM, EPSLON
      LOGICAL MATZ
      EXTERNAL F02BJF

      A2 = (X/2.0D0)**2
      EA2= (X**2) * EPSLON
      R(1) =RALY1
      R(2)= RALY2
      N2  = 2*N
      N3  = 3*N
      N4  = 4*N
      N5  = 5*N
      N6  = 6*N
***
***   The identity matrix
***
      DO 2 I=1, N
        DO 1 J=1, N
         IF(J .EQ. I)THEN
           XI(I,J)=1.0D0
         ELSE
           XI(I,J)=0.0D0
         ENDIF
 1      CONTINUE
 2    CONTINUE
***
***   Calling subroutines to find the derivative matrix (D) and
***   the second derivative (D2)
***
      CALL DERIVE (N, D)
      CALL PROD (N,D,D,D2)
***
***   Building the matrices A and B and applying the boundary
***   conditions
***
      DO 4 I=1, N
        DO 3 J=1, N
          V(I,J)= 4.0D0 * (D2(I,J)-A2 * XI(I,J))
 3      CONTINUE
 4    CONTINUE

      DO 6 I=1, N
        DO 5 J=1, N
         VN(I,J)=V(I,J)-N1 * XI(I,J)
 5      CONTINUE
 6    CONTINUE
```

```
      DO 47 K=1, 100
       IF (K.GT.2)THEN
       R(K)=(R(K-2) * SR(K-1) - R(K-1)* SR(K-2))/(SR(K-1)-SR(K-2))
       ENDIF

        RA2= (DSQRT(R(K)) * X * X) * (-1.0D0)

        DO 8 I=1, N6
          DO 7 J=1, N6
            A(I,J)=0.0D0
            B(I,J)=0.0D0
 7        CONTINUE
 8      CONTINUE
***
***   Writing the elements A11 and B11
***
        DO 9 I=1,N
          A(1,I)= 1.0D0
          A(2,I)=(-1.0D0)**(I-1)
 9      CONTINUE
        DO 11 I=3, N
          DO 10 J=1,N
           A(I,J)=VN(I-2,J)
           B(I,J)=XI(I-2,J)
 10        CONTINUE
 11     CONTINUE
***
***   Writing the elements A22 and B22
***

        DO 12 I=1,N
          A(N+1,I+N)= 2.0D0 * DBLE((I-1)**2)
          A(N+2,I+N)=(2.0D0 * DBLE((I-1)**2))*(-1.0D0)**(I)
 12     CONTINUE

        DO 14 I=N+3, N2
          DO 13 J=N+1, N2
            A(I,J)=V(I-N-2,J-N)
            B(I,J)=PM * XI(I-N-2,J-N)
 13       CONTINUE
 14     CONTINUE
***
***   Writing the elements A33 and B33
***

        DO 15 I=1,N
          A(N2+1,I+N2)=0.0D0
          A(N2+2,I+N2)=0.0D0
 15     CONTINUE

        DO 17 I=N2+3, N3
          DO 16 J=N2+1, N3
            A(I,J)= VN(I-N2-2,J-N2)
            B(I,J)= XI(I-N2-2,J-N2)
 16       CONTINUE
 17     CONTINUE
```

```
***
***   Writing the elements  A44 and B44
***

      DO 18 I=1, N
        A(N3+1,N3+I)= 1.0D0
        A(N3+2,N3+I)= (-1.0D0)**(I-1)
 18   CONTINUE
        DO 20 I=N3+3, N4
          DO 19 J=N3+1, N4
            A(I,J)=V(I-N3-2,J-N3)-(EA2 * XI(I-N3-2, J-N3))
            B(I,J)=PM * XI(I-N3-2,J-N3)
 19       CONTINUE
 20     CONTINUE
***
***   Writing the elements  A55 and B55
***

      DO 22 I=N4+3, N5
        DO 21 J=N4+1,N5
          A(N4+1,J)= 1.0D0
          A(N4+2,J)=(-1.0D0)**(J-N4-1)
          A(I,J)    =V(I-N4-2,J-N4)
          B(I,J)    =PR * XI(I-N4-2,J-N4)
 21     CONTINUE
 22   CONTINUE
***
***   Writing the element A66
***
      DO 24 I=N5+3, N6
        DO 23 J=N5+1, N6
          A(N5+1,J)=1.0D0
          A(N5+2,J)=(-1.0D0)**(J-N5-1)
          A(I,J)    =V(I-N5-2,J-N5)
 23     CONTINUE
 24   CONTINUE
***
***   Writing the element  B34
***
      DO 26 I=N2+3, N3
        DO 25 J=N3+1, N4
          B(I,J)= (-2.0D0) * PM * D(I-N2-2,J-N3)
 25     CONTINUE
 26   CONTINUE
***
***   Writing the element   A12
***
      DO 28 I=3, N
        DO 27 J=N+1, N2
          A(I,J)=2.0D0 * D(I-2,J-N)
 27     CONTINUE
 28   CONTINUE
***
***   Writing the element  A16
***
```

```
      DO 30 I=3, N
        DO 29 J=N5+1, N6
         A(I,J)=2.0D0 * DSQRT(T) * D(I-2, J-N5)
29      CONTINUE
30    CONTINUE
***
***   Writing the element  A21
***
      DO 32 I=N+3, N2
        DO 31 J=1, N
         A(I,J)= 2.0D0 * Q * D(I-N-2,J)
31      CONTINUE
32    CONTINUE
***
***   Writing the element  A31
***
      DO 34 I=N2+3, N3
        DO 33 J=1, N
         A(I,J)=(-2.0D0) * DSQRT(T) * D(I-N2-2,J)
33      CONTINUE
34    CONTINUE
***
***   Writing the element  A35
***
      DO 36 I=N2+3, N3
        DO 35 J=N4+1, N5
         A(I,J)= RA2 * XI(I-N2-2,J-N4)
35      CONTINUE
36    CONTINUE
***
***   Writing the element  A36
***
      DO 38 I=N2+3, N3
        DO 37 J=N5+1, N6
        A(N+1,J)= 2.0D0 * DBLE((J-N5-1)**2)
        A(N+2,J)=(2.0D0 * DBLE((J-N5-1)**2))*(-1.0D0)**(J-N5)
        A(I,J)= (-4.0D0) * Q * D2(I-N2-2,J-N5)
37      CONTINUE
38    CONTINUE
***
***   Writing the element  A46
***
      DO 40 I=N3+3, N4
        DO 39 J=N5+1, N6
         A(I,J)= 2.0D0 * Q * D(I-N3-2,J-N5)
39      CONTINUE
40    CONTINUE
***
***   Writing the element  A56
***
      DO 42 I=N4+3, N5
        DO 41 J=N5+1, N6
         A(I,J)=DSQRT(R(K)) * XI(I-N4-2,J-N5)
41      CONTINUE
42    CONTINUE
```

```
***
***   Writing the element   A63
***
      DO 44 I=N5+3, N6
        DO 43 J=N2+1, N3
         A(I,J)=(-1.0D0) * XI(I-N5-2,J-N2)
 43     CONTINUE
 44   CONTINUE

      MATZ = .FALSE.
      EPS1 =(10.0D0)**(-15)
      IFAIL= 0
***
***   Calling nag routine to solve the eigenvalue problem
***
     CALL F02BJF(N6,A,L,B,L,EPS1,ALFR,ALFI,BETA,MATZ,Z,L,ITER,IFAIL)

      M=0
      DO 45 I=1, N6
        IF(BETA(I) .NE. 0.0D0) THEN
          M=M+1
          ELANDAR(M)=ALFR(I)/BETA(I)
          ELANDAI(M)=ALFI(I)/BETA(I)
        ENDIF
 45   CONTINUE
***
***   Choosing the largest real part of the eigenvalue
***
      ALARGR=ELANDAR(1)

      DO 46 I=2, M
        IF ( ELANDAR(I) .GE. ALARGR) THEN
          ALARGR= ELANDAR(I)
          ALARGI= ELANDAI(I)
        ENDIF
 46   CONTINUE

      SR(K)=ALARGR
      SI(K)=ALARGI
      IF (K.GE.2) THEN
        DF=DABS(R(K)-R(K-1))
        IF(DF .LT. TOL) GOTO 48
        ENDIF
 47   CONTINUE
 48   RALY= R(K)
      SEGMAR=SR(K)
      SEGMAI=SI(K)

      RETURN
      END
```

```
***
***   Subroutine to construct the derivative matrix
***
      SUBROUTINE DERIVE (N, D)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION D(L,L)
      DO 10 I=1,N
        DO 10 J=1,N
          IF((J .GT. I) .AND. (MOD(J-I,2) .EQ. 1)) THEN
            D(I,J)=2.0D0 * DBLE (J-1)
          ELSE
            D(I,J)=0.0D0
          ENDIF
 10   CONTINUE
      DO 20 I=1,N
        D(1,I)=0.5D0 * D(1,I)
 20   CONTINUE
      RETURN
      END
***
***   Subroutine to multiply two matrices
***
      SUBROUTINE PROD (N,A,B,C)
      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
      PARAMETER (L=30)
      DIMENSION A(L,L), B(L,L), C(L,L)
      DO 20 I=1,N
        DO 20 J=1,N
          C(I,J)=0.0D0
          DO 10 M=1,N
            C(I,J)=C(I,J)+A(I,M) * B(M,J)
 10       CONTINUE
 20   CONTINUE
      RETURN
      END
```

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